# Monetary Policy Rules in an Estimated DSGE Model with Financial Frictions

*- Bachelor Thesis -*

Lukas Andreas Esser

B.Sc. Economics and Business Administration
Goethe-University Frankfurt a.M.

Supervised by Prof. Dr. Meyer-Gohde
Chair of Financial Markets and Macroeconomics

# Content

## List of Figures



## List of Tables

## 1. Introduction

The global financial crisis of 2007 led to a deep recession in the world economy and revealed weaknesses and shortcomings in the financial supervision strategy (Crotty 2009). The main reproach is that policymakers have ignored financial systemic risk (Angelini et al. 2011). Indeed, most countries had adopted an inflation targeting approach which implied that monetary policy "should focus solely on macroeconomic developments and largely ignore financial booms" (Filardo and Rungcharoenkitkul 2016, p. 3)[1]. This attitude was strengthened by the belief that differentiating between fundamental-driven asset price movement and bubbles in real time was too difficult (IMF 2015). The Central banks' task was to only react to asset prices and financial circumstances in the case they posed a direct thread to the inflation and output targets. This approach is often called "cleaning up" as monetary policy responds to the macroeconomic consequences of financial risk only when they have materialised.

However, financial stability was targeted by a prudential supervision framework. Yet, the supervision strategy focused primarily on microprudential regulation that seeks to minimise institutions' individual risk (Borio 2011). Aggregate risk that arises due to the behaviour of all institutions was not targeted appropriately.

In the European Monetary Union (EMU), these developments led to rising financial imbalances particularly in the so called GIIPS countries (Greece, Ireland, Italy, Portugal and Spain) as Quint and Rabanal (2014a) point out: These countries faced overly high credit growth rates stimulated by a low level of interest rates, GDP growth rates that were above the countries' potential as well as a massive increase in asset prices, especially real estate prices. When the crisis hit these countries, the resulting depression led to a long-standing crisis of the EMU and it took the resources of several countries to prevent Greece from leaving the monetary union.

In the aftermath of the crisis, Gerali et al. (2010) confirm that financial shocks are an important source of business cycle fluctuations in the EMU. Moreover, Claessens et al. (2009) and Jorda et al. (2013) demonstrate that financial crises amplify the business cycles, particularly the burst side of it, and they are followed by slower recoveries. Moreover, several studies investigate the role of the risk-taking channel of monetary policy: Dell'Ariccia et al. (2014) show that a low interest rate environment may result in higher leverage and more risk in the financial sector. Besides, "an easier policy can compress risk premia and push asset prices above levels justified by fundamentals" (Filardo and Rungcharoenkitkul 2016, p. 4).

[1] In contrast to industrialised countries, a lot of emerging countries included financial stability considerations in their monetary policy (see IMF 2015). Industrialised countries that took financial stability into account were Australia, Norway and Sweden.

To tackle these problems, some papers suggest the so called ‘leaning against the wind’ (LAW) approach to monetary policy: Central banks should react to the presence of rising financial imbalances by setting a higher interest rate than adequate in order to reach inflation and output goals. Contrary to the pre-crisis policy LAW reacts to financial imbalances before they burst. That is why the current discussion is often called “lean versus clean” (IMF 2015). Current literature has not come to a common consensus yet whether LAW is desirable. Several analyses demonstrate that LAW can be welfare enhancing (for instance, Curdia and Woodford 2010, Filardo and Rungcharoenkitkul 2016 and Verona et al. 2017), however, other papers show that LAW does not lead to superior welfare outcomes (For instance, Svensson 2017, IMF 2015, Cairo and Sim 2020).

Regarding the prudential framework, macroprudential tools that focus on aggregate financial risk were introduced in order to decrease the built-up of financial imbalances (FSB-BIS-IMF 2011). Considerably important in this context are the new regulation standards of Basel III.

This paper is part of a growing strand of literature that evaluates different monetary policy and macroprudential regimes using DSGE models. Special attention is paid to the question whether LAW policies are not only able to enhance welfare but also to substitute macroprudential policies. The literature can be divided into papers that employ ad-hoc loss functions and papers that use utility-based welfare criterions to evaluate different policy regimes[2].

The following papers implement utility-based welfare criterions. Quint and Rabanal (2014a) are the closest to this paper. They analyse different monetary and macroprudential policy regimes and find that a welfare maximising calibration of macroprudential policy cannot be substituted with an adjusted Taylor Rule that reacts to nominal credit growth. Moreover, the analysis points out that the use of certain macroprudential rules can affect savers’ welfare positively while decreasing the welfare of borrowers. Ozkan and Unsal (2014) apply a small open economy model with a large structural intersection to the model of Quint and Rabanal (2014a) and verify their results. In addition, they show that using both, LAW and macroprudential policy, enhances welfare even more. Benes and Kumhof (2011) employ a model that features similar financial frictions. A Taylor Rule is adjusted to include, besides inflation, the output gap and last period’s interest rate, deviations from the loan stock and the loans-to-output ratio. Their welfare analysis implies that expanding the Taylor Rule with the loan stock and loans-to-output ratio does not yield superior welfare results. Nonetheless, in the

[2] The literature that analysis monetary and macroprudential policies is far greater than the papers presented. However, only a small fraction includes monetary policy ‘leaning against the wind’ in their analysis.

presence of financial shocks, macroprudential policy modelled as countercyclical minimum capital adequacy is welfare enhancing. Kiley and Sim (2015) use credit spreads as a proxy for financial risk and demonstrate that monetary policy should not react to changes in this indicator. Macroprudential regulation can lead to higher welfare but it is confronted by important implementation challenges. Furthermore, they stress the importance of identifying the source of the shock. Medina an Roldós (2018) apply a model with an agency-cost problem in the spirit of Bernanke et al. (1999). Their results suggest that a LAW policy reacting to credit growth is not favourable. Instead, a simple Taylor Rule in combination with countercyclical reserve requirements is welfare optimising.

Other studies employ ad-hoc quadratic loss functions which is also done in this paper:

Kannan et al. (2012) use a one country model (that is the basis of the model used in this work) and implement a variety of loss functions which differ in the weights applied to output an inflation stabilising. They find that an altered Taylor Rule outperforms the combination of a Taylor Rule and macroprudential policy for all loss functions studied when the source of the shock is not known. Angeloni and Faia (2013) implement a New Keynesian model with financial risk introduced through risky investment projects. They compare different monetary policy regimes and add macroprudential regulations in the spirit of Basel II and Basel III. The results suggest that monetary policy reacting to asset prices or bank leverage in combination with anticyclical capital ratios leads to the best welfare outcome. Nuger et al. (2016) use a model with a modelling approach of financial risk similar to Angeloni and Faia (2013). The authors find out that a LAW approach can improve welfare, yet the benefits of macroprudential interventions are greater.

This paper uses the DSGE framework introduced by Quint and Rabanal (2014a). The model is a two country DSGE model of a common currency area estimated on European data. Briefly described, the framework consists of two types of agents (savers and borrowers), two goods (non-durables and durables or housing) and domestic, foreign and international credit markets. Borrowers are more impatient and willing to finance higher levels of current consumption with loans. Housing quality shocks affect the value of the housing stock and the borrowers' defaulting rate which influences lending deposit spreads. The presence of defaulting households creates financial frictions. The accelerator mechanism in the spirit of Bernanke et al. (1999) allows to study credit-driven house price bubbles.

Contrary to most of the mentioned papers, this model allows to analyse the effects of policy regimes in a monetary union. In particular, spillover effects from one country to another can be studied. The interest rate is set by the union's central bank. Macroprudential policy affects the amount of credit that banks can grant through the banks' balance sheets. Monetary policy has real effects due to the presence of sticky prices and macroprudential tools can influence the credit market conditions because of the financial frictions (Kannan et al. 2012). Moreover, the model includes monetary, financial, preference and technology shocks.

In this work the model is used to find the optimal policy mix under an ad-hoc quadratic loss function. This function features the variances of inflation, the output gap and the period change of the interest rate as well as the macroprudential tool. Every policy regime is calibrated to minimise the loss and then the value of the loss function is used to assess the performance relative to other regimes. An estimated Taylor Rule serves as the baseline scenario.

The first regime is a simple Taylor Rule that reacts to inflation, output growth and the lagged interest rate. This scenario represents a commonly used inflation targeting regime. Optimisation implies that the central bank should react stronger to inflation and the output gap compared to the baseline rule.

In a second step, the simple Taylor Rule is expanded by nominal credit growth. This rule can be regarded as the pure 'leaning against the wind' approach. In comparison to the simple Taylor Rule, the reaction to credit growth can decrease the losses further.

The combination of macroprudential policy with the simple rule represents the third policy regime. The macroprudential tool is designed to smooth credit spreads by regulating the amount that a bank can lend. In practice, this can be achieved through higher capital requirements or loan provisions (Kannan et al. 2012). In contrast to other papers, the welfare benefit of a macroprudential over the simple Taylor Rule regime policy is small and the prudential policy cannot deliver higher welfare results than LAW policies.

The last regime combines LAW with macroprudential policy. As both policy tools are optimised jointly, a cooperative policy case is modelled[3]. Optimisation results in a lower degree of macroprudential regulation than in the third regime. The additional prudential rule decreases the loss further compared to the pure LAW approach. It shows that monetary and macroprudential policies work in the same direction and that LAW policy cannot completely substitute macroprudential regulation even though a prudential approach alone is inferior to a pure LAW strategy. However, the optimal policy combination and calibration depends crucially on the source of the shock: Section 3 shows that the performance and the calibration of the regimes differ heavily depending on the source of the shock.

[3] The two policy tools can either be calibrated to maximise a common welfare criterion (cooperation) or to maximise two separate criteria (non-cooperation). Angelini et al. (2011) and Paoli and Paustian (2017) show that the distinction is important as non-cooperation can lead to welfare losses.

The remainder of the paper is organized as follows. Section 2 provides a detailed explanation of the financial accelerator mechanism and monetary and macroprudential policy rules. The optimisation results are presented and discussed in the following part 3. Section 4 goes on with a critical appraisal of the work and section 5 provides the conclusion.

# 2 Theory

## 2.1 The Model Framework

In the following, I will give a brief non-technical summary of the model introduced in Quint and Rabanal (2014a) and which is also used as the framework in this paper[4]. This chapter fully refers to Quint and Rabanal (2014a) if not stated differently. The components and mechanism of the credit market and the Financial Accelerator as well as the monetary and macroprudential policy rules will be discussed in a more detailed way in the next section. For a full explanation of the whole model and its derivation I refer to the appendix.

The model is a two-country DSGE Model of a single currency area with financial frictions. Financial frictions are introduced in the spirit of the Financial Accelerator Model of Bernanke et al. (1999), in the following BGG.

The two countries – home and foreign – represent the core and peripheral countries of the EMU. There are two types of agents: savers and borrowers. The share of savers and borrowers is the same in the two countries. Both agents consume durables (representing residential investment) and non-durables, and they supply labour for the production-process of these types of goods. In comparison to savers, borrowers have a different habit formation parameter with respect to consumption and, as they have a lower discount factor, they are more impatient and prefer earlier consumption. The distinction between savers and borrowers is the reason for credit to exist in equilibrium. This again is necessary for possible macroprudential policy interventions. Moreover, an idiosyncratic shock can affect the quality of the borrowers' housing stock which they use as collateral[5]. Borrowers default if the value of their outstanding debt is higher than the value of their housing stock. Thus, the quality shock does not only affect the borrowers' balance sheets, but it does also directly affect the defaulting rate. From this perspective, the shock can be regarded as the defaulting risk.

[4] The notation here is one-to-one taken over from Quint and Rabanal (2014a).

[5] Following Quint and Rabanal (2014a), the shock does not affect savers as this would have only small macroeconomic implication since they do not borrow.

The two goods are produced under monopolistic competition and nominal rigidities. Non-durable goods can be traded across countries, whereas durables cannot.

The credit market consists of two types of financial intermediaries: domestic and foreign financial intermediaries collect the savers' deposits and reallocate them to borrowers in form of credit or issuing bonds. International financial intermediaries buy and sell bonds between the two countries and charge a risk premium that depends on the country's net foreign asset position. Domestic and foreign financial intermediaries are risk neutral and can make profits or losses. Contrary to that, international intermediaries only make profits due to the charged risk premium. Both are owned by savers who either receive the profits or reliquidate the domestic and foreign banks in case of liquidity scarcity.

The model features financial, preference, technology and monetary policy shocks. Moreover, a unit-root technology shock is included.

In the baseline model, monetary policy rate set by the central bank for both countries follows a Taylor-Rule focussing on union-wide inflation and on real output growth.

Macroprudential policy affects the credit market through the credit volume which domestic financial intermediaries can lend to borrowers.

## 2.2 The Credit Market and the Financial Accelerator

The domestic and foreign credit markets include frictions which are modelled in the idea of the Financial Accelerator Model of BGG. As in BGG, the lending-deposit spread depends on the state of the housing market and there exists a default risk for borrowers. Nevertheless, there are important differences between the two approaches: first, there is no "costly state verification" or asymmetric information as in BGG. Consequently, borrowers will only default when they are really under water. Second, no collateral will be destroyed in the case of a default. Third, the one-period domestic (foreign) lending rate is determined via the participation constraint of the risk-neutral domestic (foreign) financial intermediary.

*Domestic Financial Intermediaries*

Deposits from savers $S_t$ are taken by domestic financial intermediaries at a deposit rate $R_t$. These deposits are combined to loans $S_t^B$, granted to borrowers who must pay a lending rate $R_t^L$. The housing stock, with value $P_t^D D_t^B$, is placed as collateral, with $P_t^D$ representing the housing price and $D_t^B$ representing the housing stock of borrowers[6].

[6] $S_t, S_t^B$ and $D_t^B$ are denoted in per-capita quantities.

Every borrower indexed by *j* is hit by a log-normally distributed idiosyncratic risk shock $\omega_t^j$, with cumulative distribution function $F(\omega)$ and $E[\omega_t^j] = 1$, which affects the value of their housing stock[7]. The actual realisation of $\omega_t^j$ is known at the end of the period and then it is common knowledge[8]. The household will only default if it is under water, meaning that its outstanding debt is higher than the value of its housing stock. Formally, that is when $\omega_{t-1}^j P_t^D D_t^B$ is smaller than $R_{t-1}^L S_{t-1}^B$. Thus, the risk shock does directly affect the household's ability to repay its loans and, consequently, it affects the defaulting rate as well. If $\omega_{t-1}^j$ is high enough, the household will simply repay the full amount of its loan, that is $R_{t-1}^L S_{t-1}^B$. If the household has to default, it has to repay the still existing value of the housing stock, $\omega_{t-1}^j P_t^D D_t^B$, to a debt-collection agency. When this is done, the defaulted household can keep the house. The agency charges a fraction μ of this payment as a commission and transfers the rest to the domestic financial intermediary. As well as the financial intermediaries, debt-collection agencies are owned by savers who receive their profits at the end of the period.

Domestic banks[9] act risk neutrally and grant one unit of credit to borrowers if the expected return is equal to the opportunity cost that is the deposit rate $R_t$. This is the bank's participation constraint.

The expected return depends on the defaulting rate and thus on the realisation of $\omega_t^j$, which is only known in period *t+1*. Consequently, in period t banks only know the ex-ante threshold value of $\omega_t^j$ denoted $\bar{\omega}_t^a$ with

$$\bar{\omega}_t^a E_t[P_{t+1}^D D_{t+1}^B] = R_t^L S_t^B. \tag{1}$$

Knowing the real threshold $\bar{\omega}_t$ which indicates the boundary between repaying or defaulting requires knowing future housing prices and the borrowers' future housing stocks. Formally, the banks' participation constraint has the following form:

$$R_t = E_t\left\{(1-\mu)G(\bar{\omega}_t^a)\frac{P_{t+1}^D D_{t+1}^B}{S_t^B} + [1-F(\bar{\omega}_t^a)]R_t^L\right\} \tag{2}$$

[7] This section tries to remain mostly non-technical. For further statistical properties of the shock, I refer to the appendix, section 7.1.1.

[8] Thus, no "costly state verification" is needed.

[9] The terms "bank" and "financial intermediary" are used as synonyms.

where $G(\overline{\omega}_t^a)$ is the expected value of $\omega_t^j$ for all values of $\omega_t^j$ below the ex-ante threshold $\overline{\omega}_t^a$, implying that $(1-\mu)G(\overline{\omega}_t^a)\,P_t^D D_t^B/S_t^B$ is the expected mean repayment the bank receives from debt-collection agencies denoted in percentage terms of outstanding debt[10]. Moreover, $[1-F(\overline{\omega}_t^a)]$ denotes the expected probability that $\omega_t^j$ is high enough for household j to completely repay its loan. In expectations, (2) must be fulfilled and thus, domestic financial intermediaries do not make profits ex-ante when they set the lending rate and the credit amount. Equation (2) is of major importance for the model as it is responsible for the accelerator effects: Suppose that the expected house prices increase. Following (2), the increase must cause a decline of the lending rate which consequently raises the borrowers' demand for housing. Thus, the house prices rise further, resulting in a credit and house price spiral. A concrete example is presented in section 3 of the paper. The effects of the mechanism are similar to credit-driven bubbles described in Mishkin (2011).

Although domestic banks cannot make profits or losses ex-ante, they can make profits or losses ex-post. Profits will be transferred to savers who own the banks and who would recapitalise the banks in case of losses.

*Foreign Financial Intermediaries*

The foreign credit market works in the same manner as the domestic market, with foreign financial intermediaries being the equivalent of domestic financial intermediaries. Consequently, the foreign lending rate $R_t^{L*}$ is set analogously to the domestic lending rate.

The domestic deposit rate is assumed to equal the risk-free rate set by the central bank. The foreign deposit rate $R_t^*$, however, does not equal the risk-free rate. It is set by international intermediaries at the international bond market.

*International Financial Intermediaries*

When domestic banks have surplus funds $B_t$ (also in per-capita quantities), they sell them to international intermediaries that will lend them to foreign banks at a rate of $R_t^*$. They set $R_t^*$ to be equal to the domestic deposit rate plus a risk premium. The spread depends on the ratio of real net foreign assets to the steady-state value of the domestic non-durable GDP. International intermediaries use the following formula to set $R_t^*$:

$$R_t^* = R_t + \left\{\vartheta_t\, exp\left[\kappa_B\left(\frac{B_t}{P_t^C Y^C}\right)\right] - 1\right\}, \tag{3}$$

[10] In other words, $G(\overline{\omega}_t^a)$ is the mean value of $\omega_t^j$ if $\omega_t^j < \overline{\omega}_t^a$. See Quint and Rabanal (2014b) for further reference.

with $B_t/P_t^C$ being real foreign assets and $Y^C$ being the steady-state value of non-durable domestic GDP. The elasticity of the risk premium is denoted $\kappa_B$, $\vartheta_t$ denotes an exogenous shock to the risk premium which can increase the spread between domestic and foreign deposit rates. When the domestic banks sell bonds on the international market, which means $B_t$ is greater than zero, foreign banks must pay a higher deposit rate than the domestic banks.
International intermediaries make positive profits equal to $(R_t^* - R_t)B_t$. They are owned by savers from both countries and each saver receives an equal share of the profits.

## 2.3 Monetary and Macroprudential Policy

*Monetary Policy*

The interest rate or risk-free rate is set by the central bank as mentioned earlier. As both countries are in a currency union, the central bank's interest rate setting directly affects the home country, as the domestic deposit rate equals the risk-free rate. It affects the foreign country through the mechanism described in equation (3).
The interest rate reacts to deviations of the union-wide inflation from its steady-state value and to the union-wide real output growth. Moreover, it depends on the last-period's interest rate. This can be interpreted as a preference for interest-rate-smoothing over time. The mechanism is also hit by an exogenous monetary policy shock.
The mentioned causal relationship between the interest rate and the different variables can be stated formally according to this adapted Taylor Rule[11]:

$$R_t = \left[\bar{R}\left(\frac{P_t^{EMU}/P_{t-1}^{EMU}}{\bar{P}_G^{EMU}}\right)^{\gamma_\pi}\left(\frac{Y_t^{EMU}}{Y_{t-1}^{EMU}}\right)^{\gamma_y}\left(\frac{S_t^{B,EMU}}{S_{t-1}^{B,EMU}}\right)^{\gamma_{cg}}\right]^{1-\gamma_R} {R_{t-1}}^{\gamma_R}\, exp(\varepsilon_t^m) \quad (4)$$

with $\bar{R}$ being the interest rate's steady-state value and $P_t^{EMU}$ the value of the union-wide consumer price index. The steady state value of the price index growth factor is denoted by $\bar{P}_G^{EMU}$[12]. The parameters $\gamma_\pi, \gamma_y$ and $\gamma_R$ represent the weights that the central bank applies to inflation, output-growth and interest-rate-smoothing. When discussing the 'leaning against the wind' approach, this simple Taylor rule is appended with the EMU-wide nominal credit growth, $S_t^{B,EM} \;/S_{t-1}^{B,EMU}$, as proposed by Christiano (2007) among others. Credit growth is chosen

[11] Taylor (1993) develops based on the work of Bryant et al. (1993) a simple rule to model the US-interest rate. This rule became very popular in monetary policy analysis as it is simple to model and it is able to reproduce the actual interest-rate setting of several central banks very precisely (see for instance Woodford 2001).
[12] The notation of the growth factor is different to Quint and Rabanal (2014a) who use $\bar{\Pi}^{EMU}$. Contrary, I will denote the inflation rate with $\Pi$.

because the Basel III regulation includes the policymakers to intervene in case of excessive credit growth (see Basel Committee on banking Supervision 2011). Moreover, recent literature shows "that excessive credit is the best predictor of financial crises in the current financial era" (Verona et al. 2017, p. 189). The parameter $\gamma_{cg}$ is measuring the strength of the interest rate's reaction to nominal credit growth. For $\gamma_{cg} = 0$ credit growth cancels out and a non-altered Taylor Rule setting can be studied.

*Macroprudential Policy*

The macroprudential policy tool is introduced in order to effect the balance sheets of domestic financial intermediaries. The balance sheet of a domestic bank (and equivalently for a foreign bank) is given by:

$$n\,\lambda \frac{1}{\eta_t}(S_t - B_t) = n(1-\lambda)S_t^B \tag{5}$$

whereby $n$ is the size of the domestic country ($1-n$ is the foreign country's size) and λ is the mass of savers ($1-\lambda$ is the mass of borrowers) – with λ having the same value in both countries. The macroprudential policy instrument is denoted $\eta_t$. Changing this instrument limits or broadens the amount of loans that banks can lend to borrowers. A higher value of $\eta_t$ represents a tighter macroprudential policy as it reduces the amount banks can lend at a given level of deposits and foreign bonds. When combining equation (2) and (5), the relationship between the lending-deposit spread and the macroprudential tool can be derived:

$$\frac{R_t^L}{R_t} = E_t\left\{\frac{\eta_t}{\frac{(1-\mu)G(\bar{\omega}_t^a)}{\bar{\omega}_t^a}+[1-F(\bar{\omega}_t^a)]}\right\}. \tag{6}$$

Equation (6) shows that a higher value of $\eta_t$ results in a higher lending-deposit spread. The intuition behind this is that equation (2) still has to hold, and the opportunity costs of the bank's funds are still the same as they can sell all their funds to international intermediaries at rate $R_t$. A higher value of $\eta_t$ leads to a lower amount of credit the banks can lend, yet the expected profit has still to be equal to $R_t$. Thus, banks have to charge a higher lending rate to ensure that the participation constraint is satisfied, c. p. The higher lending rate in turn decreases the borrowers' disposable income and the demanded credit amount.

The case in which $\eta_t$ equals one represents the scenario without macroprudential policy. The exact value of $\eta_t$ is set using the following equation:

$$\eta_t = (\Upsilon_t)^{\gamma_\eta}. \qquad (7)$$

In this formula, $\Upsilon_t$ denotes an indicator that represents deviations from steady-state values of a certain model variable. In this analysis, nominal credit growth is used. The macroprudential policy tool reacts to $\Upsilon_t$ whereby the strength of this reaction depends on $\gamma_\eta$ which is set in order to minimise a loss or to maximise a welfare function. For the foreign country, $\eta_t^*$ is designed analogously. Quint and Rabanal (2014a) force $\gamma_\eta$ and $\gamma_\gamma^*$ to be equal when modelling macroprudential policy conducted by a union-wide institution such as the central bank. Note that even though the sensitivity parameters are supposed to be the same, $\eta_t$ and $\eta_t^*$ may differ. This approach ensures that macroprudential regulation in the home country does not change after a shock in the foreign country. Instead, an housing preference shock in the foreign country increases the nominal credit growth and, consequently, $\eta_t^*$ rises. However, as foreign output and inflation deviate positively from the steady state, the interest rate increases leading to negative effects on the home country. In theory, a response of $\eta_t^*$ can partly substitute the interest rate rise and, thus, it may dampen the negative spillover effects to the home country. Hence, macroprudential policy is expected to compensate for the disadvantages of a union-wide monetary policy. A more limited approach would be to assume that using (7) with an indicator variable that represents a union-wide variable may also model this situation. In this case, $\eta_t$ is applied to both countries. However, this paper adopts the first approach.

## 2.4 Theoretical Foundations of Macroprudential Policy

*Necessity of Macroprudential Policy*

Macroprudential policies are part of several policies that aim to protect financial stability[13]. Table 1 presents a classification of exemplary selected instruments.

In the decades after the 1980s and before the crisis of 2007 had started, policymakers mainly ignored macroprudential policies when trying to sustain the stability of financial markets (Blanchard et al. 2010). Meanwhile, financial supervision focused mainly on microprudential regulation instead of paying attention to possible systemic risks in the financial sector. The idea

[13] There is no clear definition of financial stability and financial risk. Of course, both terms are connected to each other. One view suggest that financial stability implies that the financial system is able to efficiently allocate capital so that individuals can smooth their consumption over time (see Kahou and Lehar 2017). Therefore, financial risks pose a threat to financial stability and the functioning of the financial markets.

**Table 1:** *Classification of tools to promote financial stability*

| | Objective | Instruments (exemplary) |
|---|---|---|
| Macroprudential | Limit financial system-wide (systemic) distress | countercyclical capital charges, forward-looking provisions, LTV caps |
| Microprudential | Limit distress of individual institutions | Quality/quantity of capital, leverage ratio |
| Monetary Policy | Price stability | Policy rate, standard repos |
| | Liquidity management | Collateral policies, interest on reserves |
| | Lean against financial imbalances | Policy rate, reserve requirements, FX reserve buffers |
| Fiscal policy | Manage aggregate demand | Taxes, automatic stabilisers |
| | Build fiscal buffers in good times | Measures to reduce debt levels |
| Capital controls | Limit system-wide currency mismatches | limits on foreign exchange positions |
| Infrastructure policies | Strenghten the resilience of the the infrastructure of the financial system | move derivative trading on exchanges, early warning exercise |

**Source:** adapted from Caruana (2010), Hannoun (2010) and Galati and Moessner (2013).

was that "the whole financial system is sound if and only if each institution is sound" (Borio 2011, p.88). However, this approach might overlook risks which are of no importance for individual institutions but exist in the aggregate market (Brunnermeier et al. 2009). Borio (2003) describes the different attitudes towards risks as follows: Microprudential regulation considers risks to be exogenous and not to depend on the decisions made by individual institutions. In contrast to that, macroprudential regulation regards risks to be endogenous and to depend on the collective behaviour of individual institutions.

The occurrence of several financial crises during the last decades, including the 2007 crisis, has shown the failure of the regulatory system with its pure focus on microprudential policies that target individual financial distress, as well as the need for further regulatory instruments to address the systemwide financial distress or rather reduce the volatility of asset prices[14] (for instance, see Crockett 2000, Borio 2003, Caruana 2010 or Galati and Moessner 2013). This is stressed by research on the coincidence of financial crunches and busts and economic recessions done by Claessens et al. (2009). They find that 81 out of 122 investigated recessions are linked with at least one or more of the following financial market crises: credit crunches, house price busts or equity price busts. Furthermore, they show that recessions associated with one of these crises tend to be longer lasting and deeper than other recessions.

[14] Generally, there are some arguments against a full microprudential regulation framework: Crockett 2000 claims that too much microprudential regulation could harm the efficiency of the financial sector. Schinasi 2005 adds that a strong regulation on the micro-level could prevent institutions from failing. However, what is supposed to be good, could lead to a less vigorous financial system as it could damage the process of "creative destruction" which is a key element of a healthy economy.

It must be stressed that it does not mean microprudential policy should be abandoned. Caruana (2010) emphasises that macroprudential policies alone cannot safeguard financial stability either. Micro- and macroprudential policies should rather be seen as complements than as substitutes (IMF 2011).

*Substitutability Through an Altered Taylor-Rule*

The analysis and discussion of the 2007 crisis led to the common consensus that there is the need to strengthen instruments that address systemic risk in the financial sector. Table 1 presents a whole set of instruments and policies constructed to secure the overall stability of the financial system. Every group of tools is used for a specific purpose. To address systemic risk, macroprudential tools are suggested frequently in the current discussion. Nevertheless, current literature is still uncertain about how effective these tools are as the risk-talking channels are diverse and not completely comprehended yet (Filardo and Rungcharoenkitkul 2016). Therefore, some economists also consider the so called 'leaning against the wind' approach to monetary policy[15].

Monetary policy can affect financial stability through several channels. IMF (2015) differentiates between short- and medium-term effects: In the short-term, agents are not able to adjust their balance sheets in response to an increased interest rate. Thus, borrowing costs rise and decrease the profitability of firms, the households' earnings and asset prices. As a result, the financial situation of households and firms is diminished which could lead to a higher number of defaults. All in all, IMF (2015) assesses the short-term effects of an increased interest rate as a worsening of financial stability. Nonetheless, in the medium-term, agents adapt their behaviour and adjust their balance sheets. Dell'Ariccia et al. (2014) show that a medium-term implication can be a reduction in leverage. Rajan (2006) states that a low interest rate environment increases the procyclicality of financial risk. He argues that a higher interest rate also decreases the degree of aggregate risk in financial markets.

The framework described above models this approach by setting $\gamma_{cg}$ greater than zero in equation (4). This implies that the interest rate reacts to nominal credit growth. In the steady-state, nominal credit growth is equal to one. If credit growth is greater than one, e.g. due to a risk shock, the interest rate will rise. Note that an increase in interest rate will affect the banks' participation constraint (6) and banks have to set a higher lending rate so that the constraint is satisfied. The further transmission is equivalent to the transmission of the macroprudential tool

[15] Early contributions were made by Borio and Lowe (2002) among others. Woodford (2012) builds a model that incorporates financial stability considerations in an inflation targeting approach.

also described above. From the model perspective, both tools can affect credit growth. The main differences are the side-effects of an increase in the interest rate. A change in the macroprudential tool does only influence the banks' participation constraint whereas a change in the interest rate has a broader effect on the whole economy. Whether a monetary response is able to substitute for macroprudential policy depends on the exact parameterisation of the model.

There is an ongoing discussion about whether monetary policy should systematically react to financial imbalances at all. Obviously, if monetary policy is not allowed to follow financial stability goals, it cannot substitute macroprudential policies. I will therefore give a brief overview of the main contentious issues of this general discussion.

Opponents of the approach present mainly three arguments against the use of monetary policy for this purpose:

First, growing financial imbalances or asset price bubbles are difficult to detect. Tightening monetary policy as a reaction to a false identified asset bubble might lead to lower economic activity without addressing an urgent thread to the economy (Galati and Moesnner 2013). Moreover, monetary policy actions can harm the role of financial markets in allocating resources.

Second, Svensson (2017) claims that the effects of a tightening of the interest rate by a central bank are ambiguous and that the calibration of the interest rate adjustment could be difficult. Small changes in the interest rate could be insufficient, whereas a too strong increase may lead to a burst of the bubble and to higher economic damage (for instance, see Gruen, Plumb and Stone 2005). In other words, it means that "bubbles are departures from normal behaviour, and it is unrealistic to expect that the usual tools of monetary policy will be effective in abnormal conditions" (Mishkin 2008, p.3).

Third, it is argued that adding financial stability to the central bank's goals contradicts the 'Tinbergen principle', which says that every instrument should only have one policy goal to achieve (Tinberg 1952). In the concrete case, the focus on financial stability could interfere with a constant level of inflation and a small output gap.

To put in a nutshell, the opponents of the LAW-approach conclude that monetary policy reacting only to the inflation and output outlook is likely to achieve superior outcomes.

However, representatives of the approach argue against these three main arguments:

Regarding the first argument, Woodford (2012) states that it is not necessary for a central bank to exactly identify a bubble. It should rather be focused on the "set of conditions that are likely to generate significant strains in the financial system" (Borio and Lowe 2002, p.26). Identifying the circumstances that increase the risk of a crisis is easier than identifying bubbles. Borio and Drehmann (2009) introduce such an indicator that has predictive value. Woodford (2012) names high levels of leverage and high degree of maturity transformation as possible threads. Consequently, LAW-policies do not mean that asset price bubbles should be "pricked". Instead, central banks should use the interest rate to avoid circumstances under which bubbles can arise. Under this monetary policy regime, the market's reaction seems to be less uncertain as stated in the second argument. Woodford (2012) claims that small interest rate changes may have a strong influence on the institutions' behaviour regarding leverage and maturity transformation. Concerning the "Tinbergen-principle", Woodford (2012) argues that monetary policy caring about financial stability does not imply that macroprudential tools should also not be used. In contrast, the fact that macroprudential tools cannot fully address systemic risks forces central banks to use the interest rate in order to contribute to financial stability. He asserts that the trade-off between these three policy goals is not different from the trade-off between inflation and stabilising the output gap.

# 3. Application

## 3.1 Welfare criterion

Contrary to the welfare function used by Quint and Rabanal (2014a)[16], this paper employs an ad-hoc quadratic loss function as a welfare criterion to evaluate the different policy rules for several reasons: First, central banks set goals in terms of inflation or output and not in terms of social welfare. The ECB's top priority is price stability, as a secondary goal the institution aims to avoid excessive fluctuations in output and employment if not contrary to the primary goal. Thus, it is reasonable to implement a function containing inflation and output to judge different monetary policy regimes. Second, Kannan et al. (2012) argue that a household's utility-based welfare criterion presumably overestimates, in a setting with multiple types of agents, the current utility of the more impatient one.

$$L^{CB} = \sum_{t=0}^{\infty} {\beta^{CB}}^{t} \left[ (\Pi_t^{EMU})^2 + \alpha_y (Y_t^{EMU} - \bar{Y}^{EMU})^2 + \alpha_{\Delta r} (R_t - R_{t-1})^2 + \alpha_{\Delta \eta} (\eta_t^{EMU} - \eta_{t-1}^{EMU})^2 \right], \tag{8}$$

[16] They model union wide welfare as the weighted welfare of the domestic and foreign savers and borrowers. Each type's utility is measured by subtracting the utility function's steady state value from the second-order approximation of the utility function.

with $\eta_t^{EMU}$ being the country-size-weighted average of $\eta_t$ and $\eta_t^*$. The parameters $\alpha_y$, $\alpha_{\Delta r}$ and $\alpha_{\Delta\eta}$ measure the weight of the output gap, the period's interest rate and its macroprudential tool change in the loss function. $\alpha_{\Delta r}$ and $\alpha_{\Delta\eta}$ can be regarded as an inertia coefficient. The central bank's discounting factor is denoted by $\beta^{CB}$. Following Dennis and Ilbas (2016), the loss function converges to

$$L^{CB} = var\ (\Pi^{EMU}) + \alpha_y var(Y_{GAP}^{EMU}) + \alpha_{\Delta r} var(\Delta R) + \alpha_{\Delta\eta} var(\Delta\eta^{EMU})\,, \tag{9}$$

when $\beta^{CB} \rightarrow 1$. The discussion on central bank's loss functions presents several parameterisations of $\alpha_y$, which can differ to a large degree: For example, a small value of $\alpha_y = 0.048$ is introduced in Woodford (2003)[17]. The Norges Bank uses a loss function that includes a parameter value of 0.75 (Evjen and Kloster 2012, p. 4)[18]. Kannan et al. (2012) argue that the dual mandate of the Federal Reserve implies $\alpha_y$ to be equal to one. In a more recent study, Debortoli et al. (2018) derive that $\alpha_y = 1.042$ leads to welfare maximising monetary policy in the model of Smets and Wouters (2007). Although this model is estimated using U.S. data, they argue that the "transmission of monetary policy, the structure of the economy, and shocks are very similar in the European economies" (Debortoli et al. 2018, p. 2035). Consequently, their calculated weight in the loss function may also represent a good welfare approximation for models estimated on European data, such as the one used in this paper. Moreover, Evjen and Kloster (2012) argue that there is a positive relationship between financial variables indicating financial imbalances and economic activity. Thus, ascribing higher weight to the output gap incorporates a financial stability aspect in the simple loss function which could justify the high value of $\alpha_y$ from a different point of view.

The period's change in the interest rate gap as well as in the macroprudential tool are added to the loss function to model adjustment costs. Without these costs, "optimal policies will tend to generate excessive volatility in the policy rate" and, consequently, in the macroprudential tool as well (Angelini et al. 2011, p. 15). Following Dennis and Ilbas (2016), these costs enter the loss function with a relative weight to inflation of 0.5. In comparison to the literature, $\alpha_{\Delta r} = \alpha_{\Delta\eta} = 0.5$ represents a relatively high weight on the adjustment cost.

[17] The exact quarterly value presented in Woodford (2003) is $\alpha_y = 0.003$. Following Debortoli et al. (2018, p. 2023) this quarterly weight is equal to an annualised weight of 0.048.

[18] The loss function includes additional terms regarding the interest rate difference from time t-1 to t and the deviation of the interest rate from the steady state level. The weights for these two additional terms are 0.25 and 0.05.

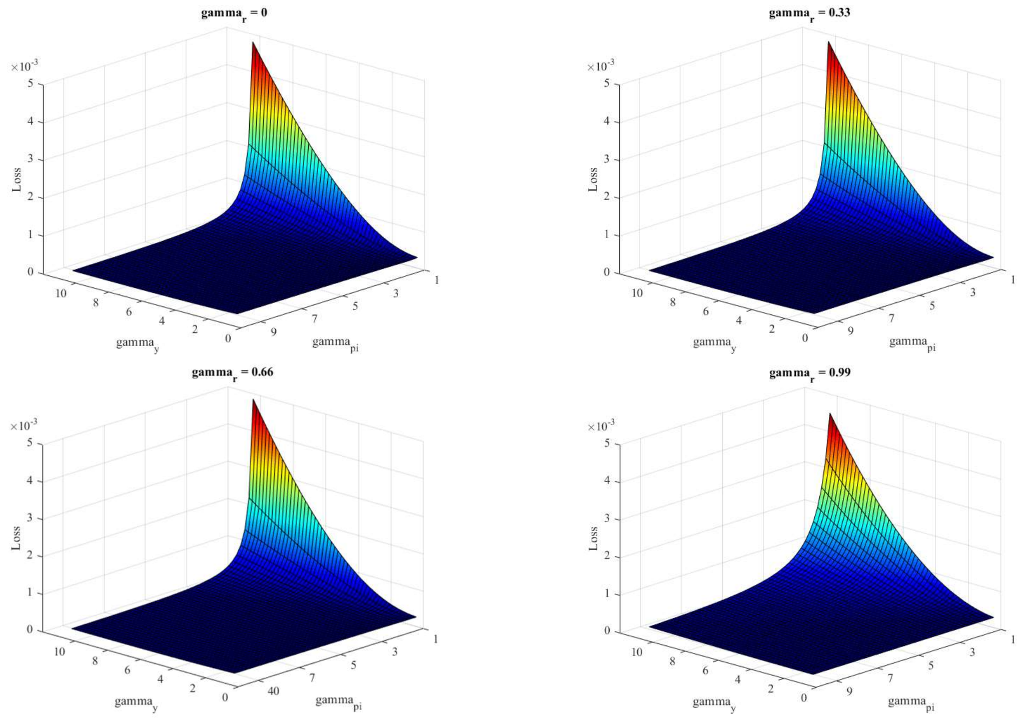
The loss function does not include any proxy variable for financial risk for mainly two reasons: First, inflation and the output gap are the main goals of monetary policy. Although the central bank reacts to financial imbalances using the interest rate or macroprudential policy, its ultimate goal is to prevent financial imbalances from disrupting stable inflation and output. Thus, it is sufficient to include the variance of inflation and the output gap (Kannan et al. 2012). Secondly, the loss function should stay as parsimonious as possible, and any additional term jeopardises this goal.

Contrary to other papers (for instance, Angelini et al. (2011), Dennis and Ilbas (2016) or Gelain and Ilbas (2017)), only one loss function is implemented. As it is assumed that the central bank is responsible for monetary and macroprudential policy, both policies are calibrated to minimise the central bank's loss[19].

## 3.2 Parameters of Monetary and Macroprudential Policy

For every of the four policy combinations analysed, the five parameters, $\gamma_\pi$, $\gamma_y$, $\gamma_{cg}$, $\gamma_R$ and $\gamma_\eta$, are set to minimise the central bank's loss function. To speed up the optimisation routine, $\gamma_y$, $\gamma_{cg}$ and $\gamma_\eta$ are restricted to be between zero and five. This is in line with the procedure of Quint

**Figure 1:** *Central bank's loss in a simple Taylor Rule environment depending on $\gamma_\pi$ and $\gamma_y$ for different fixed values of $\gamma_R$*




**Notes:** Figure 1 displays the central bank's losses depending on $\gamma_\pi$ and $\gamma_y$ for fixed values of $\gamma_R$. The graph is computed with incremental increases of 0.2.

[19] In the EU, the European Risk Board directs macroprudential policy. However, the European Central Bank controls the majority of seats and, thus, the assumption is reasonable.

and Rabanal (2014a). Figure 1 demonstrates that the benefit of allowing for greater values would be small. The parameter regarding inflation in the Taylor Rule must be greater than one and less than or equal to five[20]. Furthermore, $\gamma_R$ is restricted to be in the interval [0,1)[21].

**Table 2:** *Parameters of monetary and macroprudential policy rules in different regimes*

| | $\gamma_\pi$ | $\gamma_y$ | $\gamma_R$ | $\gamma_{cg}$ | $\gamma_\eta$ |
|---|---|---|---|---|---|
| Estimated Simple Taylor Rule | 1.558 | 0.202 | 0.802 | - | - |
| Simple Taylor Rule | 5.00 | 3.71 | 0.23 | - | - |
| Altered Taylor Rule | 5.00 | 3.70 | 0.00 | 1.23 | - |
| Simple Taylor Rule and Macroprudential Policy | 5.00 | 3.72 | 0.22 | - | 0.09 |
| Altered Taylor Rule and Macroprudential Policy | 5.00 | 3.67 | 0.00 | 1.24 | 0.03 |

**Notes:** The first row presents the estimated coefficients of the simple Taylor Rule by Quint and Rabanal (2014a). Lines two to five show the coefficients that minimise the loss function for a given policy setting unconditional on the shock.

Table 2 presents the parameter values minimising the loss function. The parameters are optimised to the second decimal place[22]. Further details on the optimisation process are provided in the appendix. The table also includes the estimated parameter of the simple Taylor Rule by Quint and Rabanal (2014a). This rule represents the baseline scenario. Moreover, this analysis follows them in not including the monetary policy shock in the simulation process, as it would mainly affect the inertia coefficient $\gamma_R$ of the Taylor Rule.

At first, the parameters differ to a large degree from the estimated rule: $\gamma_\pi$ is equal to five throughout every policy mix and more than three times as large as the estimated coefficient. The same applies to $\gamma_y$, which is frequently more than eighteen times as large as the estimated value and varies between the combinations by 0.04 at most. However, the parameter regarding inertia of the interest rate is continuously smaller and, in the case of LAW, even equal to zero. A potential reason could be that the benefits of reacting to credit growth outweigh the additional losses from higher interest rate fluctuations. The parameters differ to a larger degree, but, controlling for the use of LAW, the differences are also exceedingly small. These three parameters show a strong dissent between the actual and the optimal monetary policy under the introduced loss function in the EMU. Regarding the coefficient of the macroprudential tool, it is relatively surprising that $\gamma_\eta$ takes on far smaller values than $\gamma_{cg}$, although both react to the same indicator variable, both are implemented to dampen the effects of excessive credit growth and both influence the banks' participation constraints. As $\gamma_y$ decreases only to a small degree when including LAW, the high value of $\gamma_{cg}$ cannot fully be explained by a substitution effect

[20] The reason why $\gamma_\pi$ must be greater than one is that a smaller value would not lead to a unique equilibrium in the neighbourhood of the steady state, as the number of eigenvalues greater than one is not equal to the number of forward-looking variables.

[21] $\gamma_R$ is not allowed to be equal to one for the same reason that applies to $\gamma_\pi$.

[22] The welfare gain of further optimising becomes exceedingly small. See table 6 in the appendix.

on the output gap parameter. Generally, $\gamma_\eta$ is almost neglectable compared to the other parameters.

## 3.3 Results

To identify the best policy, the loss for every combination is computed and shown in table 3. Additionally, the table presents the unconditional variances of union-wide inflation, the output gap as well as the period's interest rate and macroprudential tool change.

**Table 3:** *Variances and welfare gain for different policy combinations*

| | Variance [x $10^{-4}$] | | | | Welfare gain |
|---|---|---|---|---|---|
| | $\Pi^{EMU}$ | $Y_{GAP}^{EMU}$ | $\Delta R$ | $\Delta\eta$ | |
| Estimated Taylor Rule | 0.10447 | 2.9994 | 0.0086048 | - | - |
| Simple Taylor Rule | 0.1664 | 2.7166 | 0.085477 | - | 6.027 % |
| Altered Taylor Rule | 0.19541 | 2.6385 | 0.065988 | - | 7.955 % |
| Simple Taylor Rule and Macroprudential Policy | 0.16723 | 2.7138 | 0.082629 | 0.0032935 | 6.088 % |
| Altered Taylor Rule and Macroprudential Policy | 0.19424 | 2.6405 | 0.063182 | 0.0003756 | 7.961 % |

**Notes:** The welfare gain is computed as the percentage decrease of the loss function compared to the baseline loss of the estimated simple Taylor Rule. The numbers in italics represent the percentage changes relative to the estimated Taylor Rule. As the macroprudential tool is not estimated, changes are not given.

First, every policy strategy decreases central bank losses by more than 6%. The best policy strategy is to adapt a LAW approach while also using macroprudential tools. This policy strategy reduces the losses by 7.961%. However, the advantage over an adjusted Taylor Rule without macroprudential policy is small. Table 3 shows that macroprudential policy is generally not able to contribute greatly to lower losses, whereas including LAW leads to a welfare gain of more than 1.8 percentage points in every situation. Nevertheless, it shows that macroprudential policy has a positive effect on the economy even though it is small. The results suggest that an altered Taylor Rule is not just able to substitute for macroprudential policy. The next section will provide further insights.

Secondly, table 3 provides insights into how every single policy goal is effected by the different policies.. Although every policy attaches a higher weight on inflation in the Taylor Rule, the inflation's variance increases severely compared to the baseline scenario, even when its size is small compared to the inflation of the output gap. In contrast, the optimised rules predict a decrease in the volatility of the output gap. As the output gap's volatility dominates the loss function due to its magnitude, the increase in the variance of inflation can partly be explained by the dominant role of output stabilisation. In the optimised scenarios, the fluctuations in the interest rate rise due to the smaller coefficient in interest rate smoothing. Moreover, the results show that introducing nominal credit growth in the Taylor Rule positively contributes to output stability.

Generally, my findings suggest that a far more aggressive monetary policy leads to better outcomes than actual policy represented by the estimated rule.

## 3.4 Interpretation

In this section, the aforementioned results are put into perspective and interpreted using impulse response functions to a housing demand shock, a risk shock and a permanent technology shock. The following figures do not include the responses of a simple and altered Taylor Rule with macroprudential policy[23].

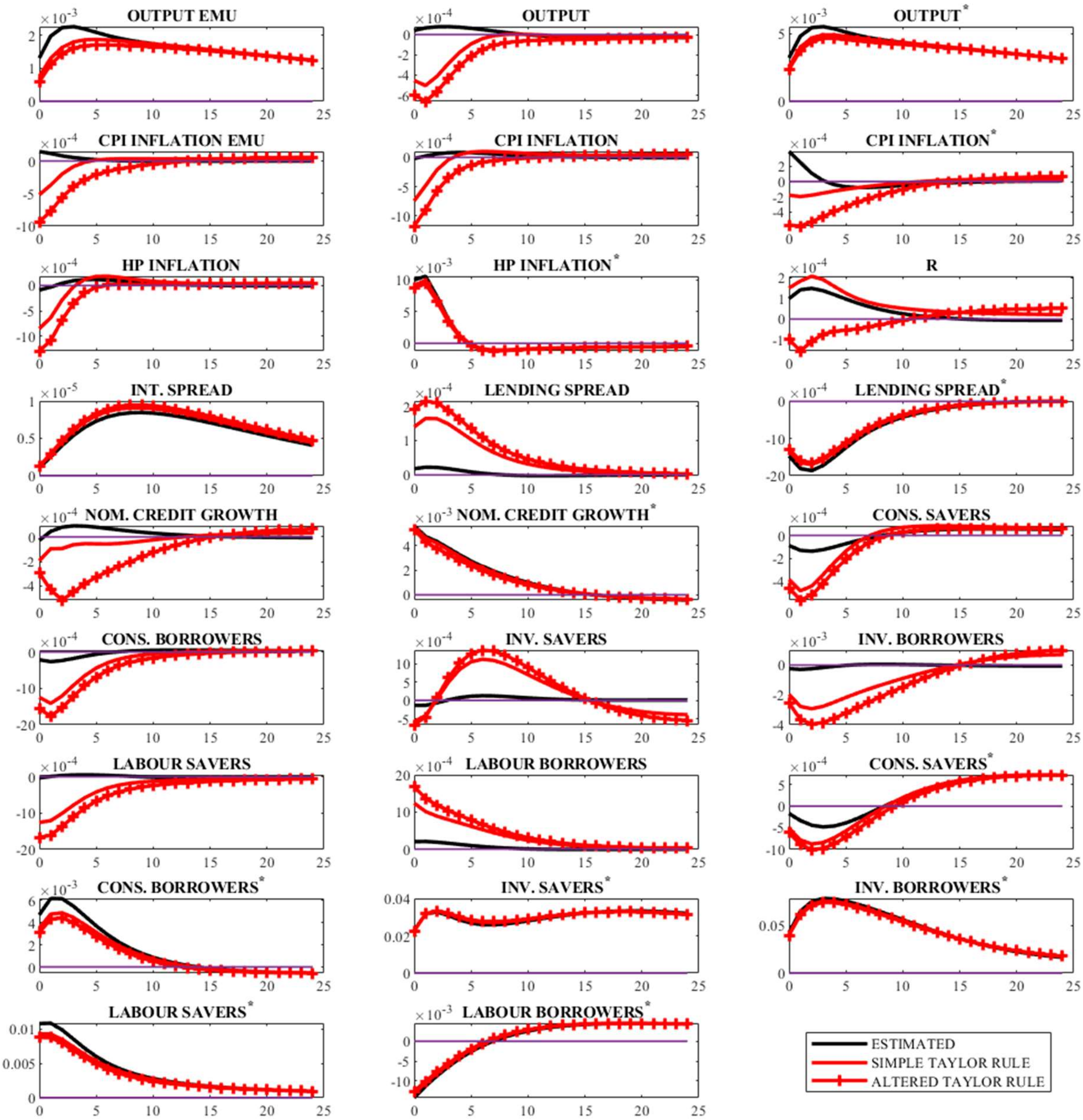
Figure 2 presents the impulse response functions of several key variables to a foreign housing demand shock which is normalised to increase the foreign housing prices (HP) by one percent. This scenario represent a foreign house price boom. The demand shock affects the marginal utility of durable consumption or housing, respectively. In a model without a Financial Accelerator Mechanism, the shock would only lead to a relatively higher share of durable compared to non-durable consumption. In this model, the higher demand for residential investment results in higher house prices. These higher house prices in turn raise the value of the borrowers' housing stock. This decreases the lending rate due to the lower default rate and increases the loan amount that borrowers can lend. That accelerator effects lower the lending rate can be seen in a decreasing lending spread. Consequently, borrowers have a greater amount of disposable income which they use to raise their non-durable consumption as well as simultaneously decreasing their amount of labour. Thus, the Financial Accelerator allows to reproduce a positive relationship between residential investment and consumption which is a feature often observed in household data (for instance, Iacoviello 2011 and de Bond and Gieseck 2020). Under the estimated Taylor Rule, the higher residential investment increases foreign output and inflation which is answered by the central bank with a modest increase in the interest rate. The higher interest rate affects the home country to a low degree, as spillover effects on home variables barely lead to numerically big fluctuations.

The optimal Taylor Rule regime reacts stronger to the positive outut gap and, consequently, a higher interest rate is set. The figure shows that the lending spread decreases to a lesser extent

[23] As both rules differ only to a exeedingly small degree from the rules without macroprudential policy, they are not presented in the figures. Otherwise they would be hard to interpret. Nonetheless, the appendix provides the impulse responses of the two rules.

than under the estimated rule and residentil investment of borrowers and savers does not change

**Figure 2:** *Impulse Response to a Housing Demand Shock in the Periphery*




**Notes:** The figure shows the impulse responses of the estimated Taylor Rule, the optimal simple Taylor Rule and the altered Taylor Rule regimes to a positive foreign housing demand shock. The regimes' calibration is displayed in table 2.

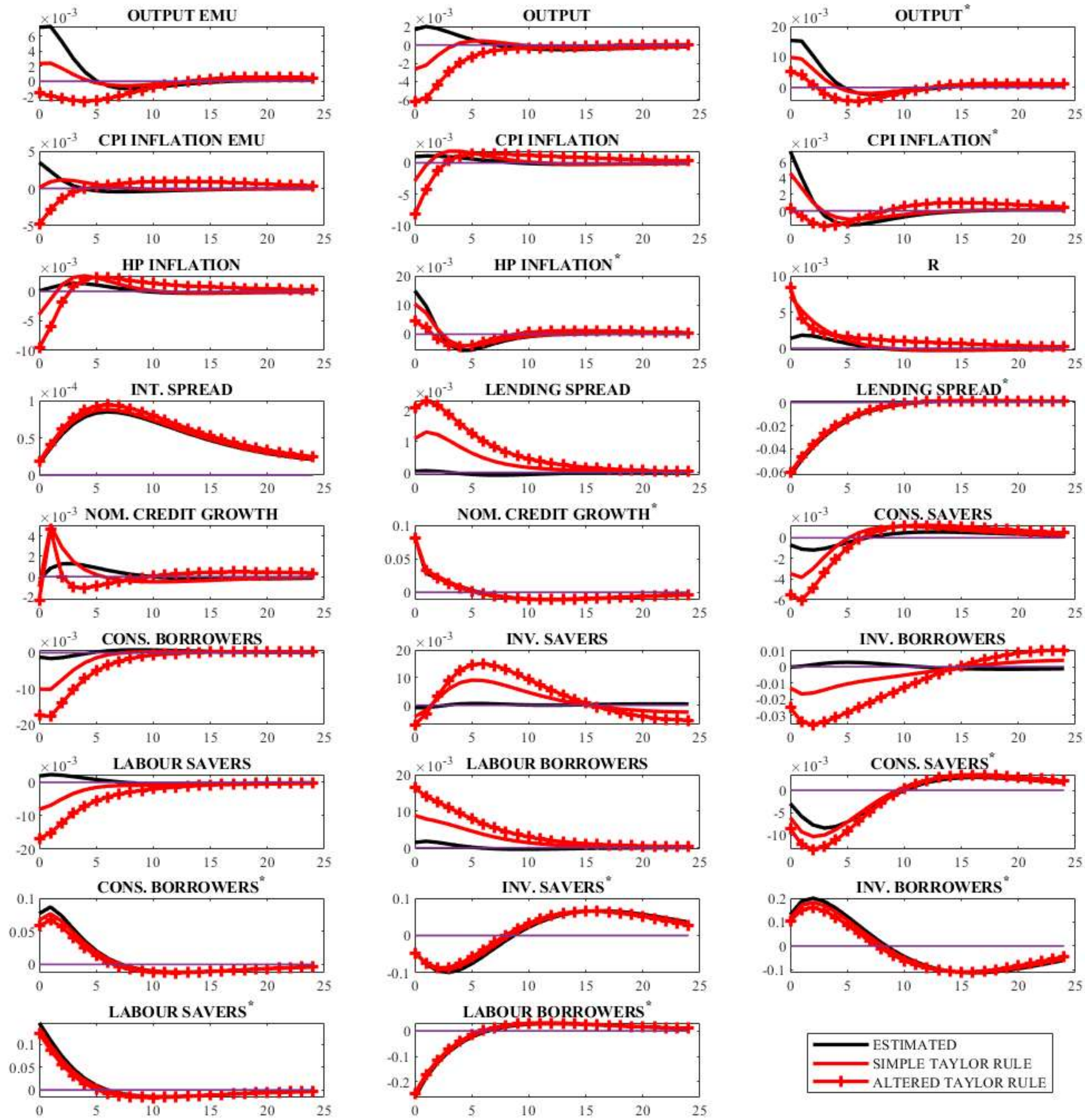
significantly. However, the optimal Taylor Rule regime is able to decrease the accelerator effects: The increse in foreign borrowers' consumption is weaker. As the rise in foreign output is primarily forced by the increased investment expenditures, the lower degree of consumption results in modest effects on output. Moreover, foreign country's inflation falls below the steady state level under this regime. This can partly be explained by a lower wage level which decreases marginal costs and thus the prices. Overall, the optimal Taylor Rule decreases the volatility of foreign varibales while the higher growth in the interest rate leads to worse spillover effects on the home economy.

Under a regime in which monetary policy reacts to changes in nominal credit gowth, the accelerator effects can be decreased even further. Thus, the rise in foreign output gap is slightly weaker than under the optimal Taylor Rule. However, contrary to first expectations, although the rule is more aggressive, the interest rate is lowered instead of raised by the central bank. This result is not uniquely found by this paper and confirmed by Kannan et al. (2012) as well as Quint and Rabanal (2014a). They explain the decrease in the interest rate with expectations effects: As the model is fully forward-looking, households take the stronger interest rate response into acount when optimising their utility. Thus, "monetary policy works through the threat of a stronger reaction, rather than by actually delivering that stronger reaction" (Kannan et al. 2012, p. 22). The LAW strategy results in the greatest decrease of inflation. It cannot be inferred which rule performs the best. This task will be examined in the following.

Figure 3 depicts the effects of a foreign risk shock that decreases the defaulting risk of foreign borrowers. It is normalised to decrease the lending-deposit spread by annualised 25 basis points. Due to the lower defaulting risk, the lending rate decreases and borrowing gets cheaper. This results in a higher level of credit granted as can be seen from the increasing foreign nominal credit growth rate. Consequently, the borrowers' higher disposable income raises the demand for residential investment and the house prices rise. This lead to the same accelerator effects as in the case of the housing demand shock. However, these accelerator effects start with delay, explaining the foreign borrowers investment behaviour: It increases in the first period due to the higher disposable income and then increases even further in the consecutive periods due to accelerator effects. Foreign borrowers' consumption follows the same pattern for the same reasons. Analysing the foreign savers' behaviour, the difference to the housing demand shock becomes apparent: Savers decrease investment following increased house prices. Besides, they face higher opportunity cost since the return on deposits rises with the interest rate. As the increase in borrowers' expenditures outweighs the decrease in the savers' consumption, foreign output and CPI inflation rise.

Moreover, Kannan et al. (2012) point out that the risk shock inhabits similarities to the financial crisis of 2007: When the shock vanishes and financial conditions return to their steady state value, residential investment of borrowers must drop under steady state levels to get the housing stock back to its natural value. The lower demand for housing results in accelerator effects with opposite side as examined above which results in a lower levels of investment and consumption and a recession that follows the expansion periods. Note that foreign borrowers are the driving factor behind the fluctuations in output and inflation. However, the fluctuations are dampened by the savers' behaviour as they respond countercyclically to the variations in prices.

**Figure 3:** *Impulse Response to a Risk Shock in the Periphery*




**Notes:** The figure shows the impulse responses of the estimated Taylor Rule, the optimal simple Taylor Rule and the altered Taylor Rule regimes to a positive foreign risk shock. The regimes' calibration is displayed in table 2.

The estimated Tayler Rule increases the interest rate to the rise in output and inflation. Both, the optimal Taylor Rule regime and the LAW regime, set a even higher interest rate which results in lower fluctuations of output and inflation. Hence, the optimal simple Taylor Rule leads to a higher output and inflation increase but to a lower home country recession as the LAW strategy. As a result, the estimated rule implies the smallest spillover effects on the home country. Under this regime, the expansion of the foreign country leads to higher demand for non-durables produced in the home country. Therefore, the home country experiences a small expansion as well. Under the two other regimes, the home country's output falls below the steady state due to the stronger monetary reaction.

Finally, on a union-wide level, the estimated rule promotes a massive expansion but leads to a recession when the influence of the shock vanishes. On the other side, a LAW policy is the best suited to reduce  foreign output and inflation fluctuations but has got the greatest spillover effects on the home country. The optimal simple Taylor Rule regime fosters the smallest fluctuations in union-wide output and inlfation.

Note that in both scenarios, LAW policies are not able to reduce the fluctuations of credit growth or the lending spread. Even though LAW is introduced to decrease aggregate financial risk, Figure 2 and 3 show that it does reduce the accelerator effects but not the indicators of financial risk. This result is also achieved by Kannan et al. (2012). However, nominal credit growth is the indicator variable for financial risk but the accelerator mechanism is responsible for a substantial part of the economic fluctuations. Thus, a LAW approach can reduce financial risk when the accelerator mechanism is recognised to be part of it.

Technology shocks are the third kind of shocks that are featured in the model. Figure 4 plots the impulse responses to a union-wide permanent technology shock[24]. The shock affects the intermediate goods producers and decreases their marginal costs. As it is a permanent shock, all real variables converge to their new steady state. Due to the symmetry of the model, the impulse responses of home and foreign varibales are the same.

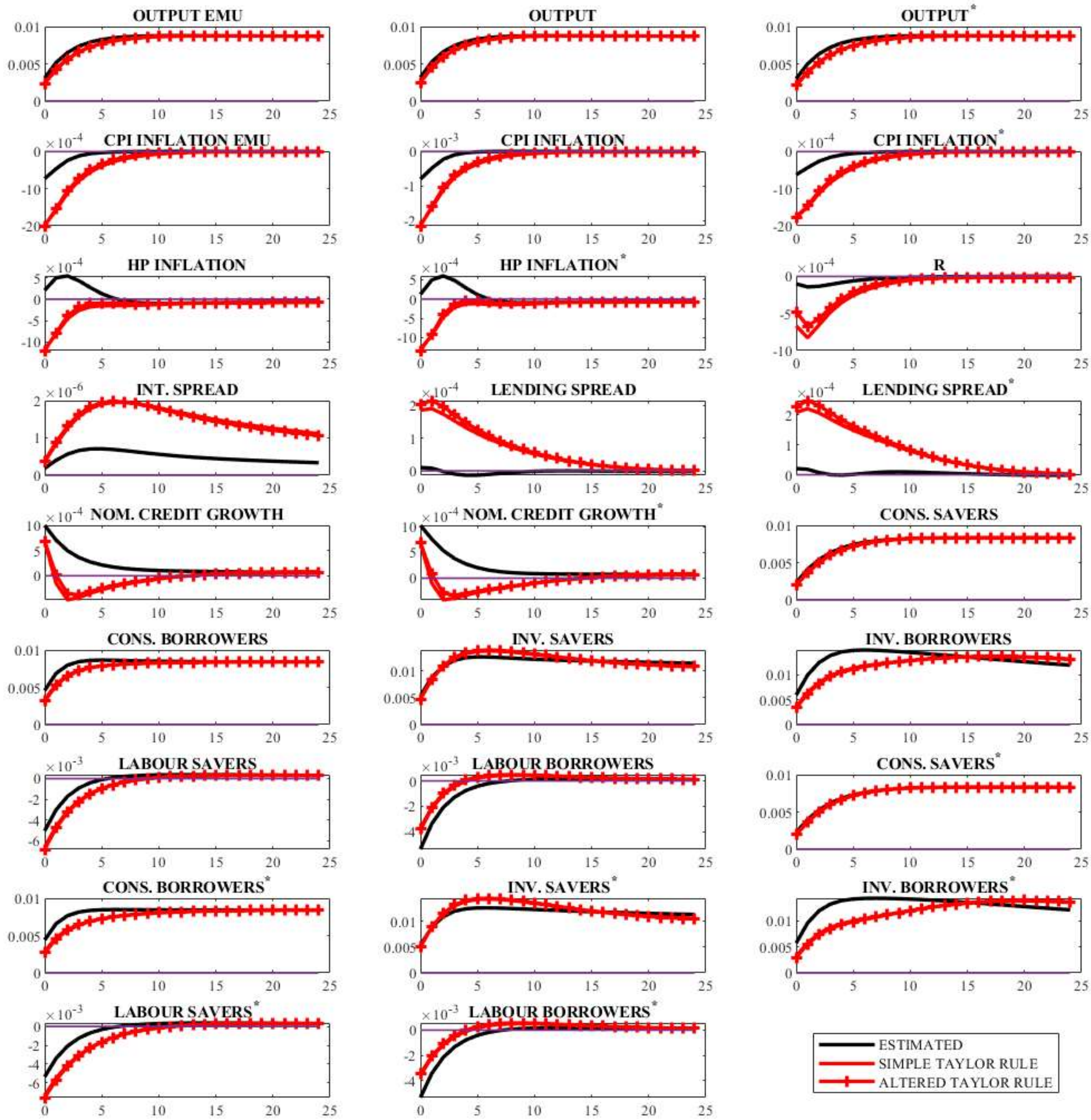
When the shock hits the economy, the steady state of real variables changes. As the level of the new steady state is higher than the previous one, all real varibles are below their new 'natural' level. Consequently, the central bank lowers the interest rate. The change under the estimated rule is the smallest but provides the fastest convergence of ouptut and inflation to the new steady state levels. This regime also leads to the least fluctuations in the lending spread and the spread beween home and foreign deposit rate. However, the optimal simple and altered Taylor Rules attach higher weight on closing the output gap and, consequently, they reduce the interest rate further than the estimated rule. Due to the increase in credit growth, the altered Taylor Rule sets a slightly higher interest rate. Inflation is under its steady state value and the lower monetary

[24] The shock's size is one-standard-deviation.

policy rate of both regimes leads primarily to even lower rates of inflation and, secondly, to a slower convergence back to the steady state.

The estimated rule delivers better results, as the two other rules ascribe too much weight on the output gap in this scenario, leading to very low levels of interest rates.

**Figure 4:** *Impulse Response to a Union-Wide Permanent Technology Shock*




**Notes:** The figure shows the impulse responses of the estimated Taylor Rule, the optimal simple Taylor Rule and the altered Taylor Rule regimes to a positive union-wide technology shock. The regimes' calibration is displayed in table 2.

The figures reveal that the ability of the different policy regimes to stabilise output and inflation, crucially depends on the source of the shock. Table 4 quantifies this ability as percentage decrease in the loss function depending on the source of the shock.

**Table 4:** *Welfare gain of the four policy regimes conditional on the kind of the shock*

| | Welfare Gain | | |
|---|---|---|---|
| | Preference Shocks | Financial Shocks | Technology Shocks |
| Simple Taylor Rule | 27.575 % | 76.89 % | - 25.446 % |
| Altered Taylor Rule | 32.018 % | 46.166 % | - 24.718 % |
| Simple Taylor Rule and Macroprudential Policy | 27.655 % | 78.873 % | - 25.532 % |
| Altered Taylor Rule and Macroprudential Policy | 31.866 % | 47.86 % | - 24.618 % |

**Notes:** The welfare gain is the percentage decrease of the loss function relative to the estimated rule. The loss is computed by simulating the model solely with the given kind of shocks.

The results show that the optimised rules are superior to the estimated rule in the case of preference and financial shocks. Moreover, they show that LAW and macroprudential policies are more beneficial in the presence of preference shocks. However, when the economy is hit by a financial shock, only macroprudential policy is advantageous while LAW policies lead to a substantial welfare loss compared to the simple Taylor Rule, which does not react to changes in nominal credit growth. If the union is hit by a technology shock, every optimised rule leads to an increase in the loss function relative to the estimated rule. The differences in the welfare loss are small, yet a LAW approach with macroprudential policy results in the lowest loss of the four optimised rules.

This exercise demonstrates that LAW and macroprudential policies are beneficial even without being able to identify the source of the shock (table 2). Still table 4 reveals that the calibration of the policy rules may differ if one is capable of identifying the source of the shock. For that purpose, the optimisation procedure is repeated but this time conditional on the source of the shock. The results are presented in table 5.

The table confirms the assumption made above. It shows that reacting to nominal credit growth with the Taylor Rule or the macroprudential tool is beneficial in the presence of preference and risk shocks. Note that in the case of financial risk shocks, the change in the interest rate due to an increase in nominal credit growth has nearly the same size as the change in the macroprudential tool[25]. I assume that a potential reason could be that the financial risk shock directly hits the banks' participation constraint. Thus, macroprudential policy can directly affect financial markets. In contrast, a preference shock influences the participation constraint with a one period lag. In the presence of risk shocks, a LAW strategy cannot replace macroprudential

**Table 5:** *Parameters of monetary and macroprudential policy conditional on the kind of a shock*

| | $\gamma_\pi$ | $\gamma_y$ | $\gamma_R$ | $\gamma_{cg}$ | $\gamma_\eta$ | Welfare Gain |
|---|---|---|---|---|---|---|
| | | | Preference Shocks | | | |
| Simple Taylor Rule | 2.27 | 5.00 | 0.28 | - | - | 35.368 % |
| Altered Taylor Rule | 2.09 | 5.00 | 0.00 | 2.27 | - | 40.072 % |
| Simple Taylor Rule and Macroprudential Policy | 2.26 | 5.00 | 0.30 | - | 0.18 | 35.499 % |
| Altered Taylor Rule and Macroprudential Policy | 2.09 | 5.00 | 0.00 | 2.27 | 0.00 | 40.072 % |
| | | | Financial Shocks | | | |
| Simple Taylor Rule | 5.00 | 5.00 | 0.61 | - | - | 79.137 % |
| Altered Taylor Rule | 5.00 | 5.00 | 0.68 | 0.32 | - | 81.890 % |
| Simple Taylor Rule and Macroprudential Policy | 5.00 | 5.00 | 0.58 | - | 0.09 | 80.492 % |
| Altered Taylor Rule and Macroprudential Policy | 5.00 | 5.00 | 0.65 | 0.29 | 0.07 | 82.629 % |
| | | | Technology Shocks | | | |
| Simple Taylor Rule | 1.01 | 0.00 | 0.79 | - | - | 12.338 % |
| Altered Taylor Rule | 1.01 | 0.00 | 0.79 | 0.00 | - | 12.338 % |
| Simple Taylor Rule and Macroprudential Policy | 1.01 | 0.00 | 0.79 | 0.00 | 0.00 | 12.338 % |
| Altered Taylor Rule and Macroprudential Policy | 1.01 | 0.00 | 0.79 | - | 0.00 | 12.338 % |

**Notes:** Shown are the optimised parameter values conditional on the source of a shock. The values can be calculated by simulating the model solely with the given shocks. The welfare gain is computed as the percentage decrease of the loss function compared to the baseline loss of the estimated simple Taylor Rule.

advantages. However, when a preference shock hits the economy, macroprudential policy will not further enhance welfare in case that monetary policy follows a LAW approach. The impulse response and the results from table 4 suggest that LAW and macroprudential policy worsen the outcomes in the case of technology shocks. Optimising the coefficients solely under this kind of shocks proves it as optimal. The ideal policy regime consists of a simple Taylor Rule that reacts to inflation only.

The sensitivity of the response to the output gap is at its upper limit regarding risk and preference shocks. The reaction of the interest rate to inflation is the strongest in the presence of risk shocks and lower in the case of preference shocks.

[25] Following equation (4), the sensitivity of the interest rate to nominal credit growth is equal to the product of $\gamma_{cg}$ and $\gamma_R$. The resulting value is very close to the sensitivity of the macroprudential tool.

The table demonstrates that optimal monetary policy differs immensely during “normal times” (only technology shocks) from periods with financial or preference shocks.

# 4. Critical Appraisal

## 4.1 Results

The introduction shows that the current discussion on the relationship between monetary policy, especially ‘leaning against the wind’, and macroprudential regulation mainly considers macroprudential tools to be superior to LAW. The results of this paper are clearly inconsistent with these findings. Only Kannan et al. (2012) come to the same conclusion and their model set-up is very similar to the one used in this analysis.

Another critical deviation from the literature is that macroprudential tools are not only inferior to a LAW strategy but almost ineffective when the source of the shock is not known. These differences are mainly driven by model-specific issues which are described in the following part. Nevertheless, the results conditional on the source of the shock are closer to the outcomes of other papers such as Quint and Rabanal (2014a).

The sensitivity parameters regarding inflation, output and interest rate smoothing are mainly in line with the literature as well. Nevertheless, there are papers such as Benes and Kumhof (2011) that set other limits for the possible range of the coefficients. The inflation sensitivity of 5 is confirmed by Kannan et al (2012) and Quint and Rabanal (2014a) but, nonetheless, such a high value is difficult to implement in central bank reality. Further robustness checks have to be done in order to confirm the results independently from the parameter cap.

However, the deductions from the quantitative results have to be curbed to some extent: This paper compares the LAW approach with only one macroprudential tool. Moreover, only one indicator variable is used. Other papers analyse different macroprudential tools and come to different conclusions[26]. The same critique applies to the use of credit growth as the indicator variable. Papers that compared the performance of several indicator variables for financial distress show that different indicators often lead to different welfare outcomes[27]. Thus, the results have to be again relativised to a certain extent: Potentially, the use of other proxy variables and macroprudential tools can change the outcome and the conclusions drawn from it. Generally, there are a lot of issues and open questions that are not dealt with in this paper. Several papers go far beyond the scope of my approach. Especially, the growing literature that

[26] The literature review from the introduction section presents papers that use other approaches and come to different conclusions.

[27] For instance, Quint and Rabanal (2014a) compare nominal credit growth and the credit-to-GDP ratio and come to different welfare results.

focus solely on macroprudential regulation provides further insights. Galati and Moessner (2013) and Kahou and Lehar (2017) present literature reviews of the current macroprudential discussion.

## 4.2 The Model Framework

As mentioned above, the goal of macroprudential policy is to limit system-wide financial distress and LAW policies target the same goal. Though, the framework does not concretely model this systemic risk. Instead, nominal credit growth is placed as a proxy for it. However, technology shocks reveal the main weakness in this approach: Referring to figure 4, a temporary increase in nominal credit growth is necessary for borrowers to reach their new steady state level of investment. In this case, interpreting credit growth as rising financial imbalances is a fallacy as the increased credit amount is covered by the sustained higher level of technology and output. This scenario is not similar to the pre-crisis situation of the GIIPS countries as their credit growth was fuelled by cheap credit (Quint and Rabanal 2014a). The example shows that a framework that concretely models systemic risk can produce more welfare enhancing calibration results and it could be able to reduce the varying performance of LAW and macroprudential policy depending on the source of the shock.

Besides, Kannan et al. (2012) comment that the modelling approach of macroprudential policy is relatively simple. It does not treat the questions how these tools are controlled and calibrated. The conclusions of this paper are primarily influenced by two factors: The structure of the model and the welfare criterion. Potential shortcomings of the model are described above. Regarding the loss function, it must be stressed that the results can vary tremendously when changing the weights or the included variables. First, the mentioned higher fluctuations of household specific variables under the optimised regimes are at least partly forced by the quadratic loss function and its strict focus on inflation and output. This will become apparent by comparing the results of Quint and Rabanal (2014a) and the results of this paper. As mentioned previously, they use a utility-based welfare criterion which results in higher volatility of output and inflation but in lower volatility of household specific variables. It shows that deductions depend crucially on the employed welfare criterion. The same conclusion can be drawn concerning the parameterisation of the loss function.

## 4.3 Extensions and Bigger Picture

The scenario evaluated in this paper can be extended in various dimensions: First, the case of union-wide macroprudential policy is examined. In possible extensions, one could study the role of nationally independent macroprudential authorities or even the combination of union-wide and national instruments. Second, this paper models monetary and macroprudential policy to be cooperative, meaning that both are set to minimise on common loss function. This simplification is justified due to the dominating position of central bankers on the European Systemic Risk Board. However, other members come from e.g. the European Banking Authority or the European Securities and Markets Authority. It is questionable whether the preferences of these members are reflected in a loss function which only minimises inflation and the output gap and does not care about financial risk. Thus, a second extension could introduce a loss function for the macroprudential authority. This in turn adds a coordination problem to the analysis similar to Angelini et al. (2011) among others. As mentioned above, other indicator variables than nominal credit growth were studied in other papers. In order to check the results for robustness, the analysis can be repeated using other proxy variables for systemic risk. Third, one can try to improve the model and its weaknesses stated above. The key challenge is to develop a better modelling approach for systemic risk. Exemplary, He and Krishnamuthy (2014) extend the baseline mechanism, namely the Financial Accelerator, and build a model that is able to match macroeconomic and financial data. Finally, this paper discussed two policy strategies to tackle financial risk. However, as mentioned above, macroprudential regulation is far more diverse in its tools than it is modelled in this paper. Further research has to analyse other available tools and their interaction as it is done by Popovan et al. (2017).

Saunders and Tulip (2019) stress that the current literature has generally weaknesses in understanding the channels through which LAW works. Hence, further research has to be done in this area to fully understand the implications of a LAW approach.

Moreover, there are other strategies available that also play an important role such as expansionary fiscal policy (IMF 2015).

In the following abstract, the results are put into perspective. What do the results imply for actual policy and where are difficulties in implementing such strategies?

A key challenge of monetary policy is how to communicate policy decisions to the public. The central bank in the EMU acts independently but nonetheless decisions are commentated and politicians as well as the public try to influence them. For instance, the low level of interest rates is frequently criticised for its negative impact on savers' deposits. What I want to demonstrate is that the high parameters of monetary and macroprudential policy are potentially difficult to justify in front of the public.

Furthermore, the findings imply a remarkable change in central bank behaviour: As the optimised rules are far more aggressive, one can doubt whether it is realistic that the ECB would adopt their behaviour.

The results suggest that extending monetary policy with financial stability concerns makes sense. However, in the long run, a LAW strategy can potentially lead to a lower level of inflation (IMF 2015). This in turn may cause lower inflation expectations and, thus, real rates and financial risk remain at the same level but the risk of hitting the zero lower bound rises.

Nevertheless, this is not the only implication of LAW policy: Caruana (2011) argues that the financial cycle is usually longer lasting than a business cycle and that financial risk can develop steadily for several years. Thus, when monetary policy aims to target financial imbalances, central banks need to prolong their policy horizons.

## 5 Conclusion

Different monetary and macroprudential policy regimes in a DSGE model of the EMU were analysed in this paper. Special attention was paid to the question whether monetary policy 'leaning against the wind' can substitute for macroprudential policy. Nominal credit growth works as an indicator for financial risk. A quadratic loss function including inflation, the output gap and the change in the interest rate and the macroprudential tool was used to evaluate the performance of each policy regime.

The results suggest that a LAW strategy with macroprudential policy is the loss minimising policy regime. However, macroprudential regulation decreases the losses only to a small degree. Nevertheless, the results also imply that LAW policy alone can partly substitute for the advantages of macroprudential policy but not fully. This paper suggests that monetary policy should react more aggressively to changes in inflation, output and nominal credit growth.

It has been shown that a central bank can further decrease the losses when the source of the shock is known. Optimal policy reactions differ to a large degree between preference, risk and technology shocks: LAW and macroprudential policies are welfare enhancing in the presence of preference and risk shocks. When the union is hit by a technology shock, both policies cannot achieve better outcomes than a simple Taylor Rule regime as they increase the countercyclical behaviour of the credit spread. The results stress the importance of central banks being able to identify the source of the shock.

The results have to be put into perspective to a certain extent as only one macroprudential intervention is studied. Moreover, a proxy variable for systemic risk was used instead of modelling systemic risk directly which is a key challenge for further research done in this area.

## 6 References


Angelini, P., S. Neri, and F. Panetta (2011): “Monetary and macroprudential policies,” Bank of Italy Working Paper, No. 801.

Angeloni, I., & Faia, E. (2013). Capital regulation and monetary policy with fragile banks. *Journal of Monetary Economics*, *60*(3), 311-324.

Basel Committee on Banking Supervision (2011): “Basel III: A global regulatory framework for more resilient banks and banking systems,” Bank for International Settlements.

Bernanke, B. S., M. Gertler, and S. Gilchrist (1999): “The financial accelerator in a quantitative business cycle framework,” in *Handbook of macroeconomics*, ed.: J. B. Taylor and M. Woodford, Volume 1C, Chapter 21, 1341-1393, North-Holland.

Benes, J., and M. Kumhof (2011): “Risky bank lending and optimal capital adequacy regulation,” IMF Working Papers, 1-27.

Blanchard, O., G. Dell’Ariccia, and P. Mauro (2010): “Rethinking macroeconomic policy,” *Journal of Money, Credit and Banking*, 42, 199-215.

de Bondt, G., Gieseck, A., and M. Tujula (2020): “Household wealth and consumption in the euro area,” *Economic Bulletin Articles* (1), 46-61.

Borio, C. (2003): “Towards a macroprudential framework for financial supervision and regulation?,” *CESifo Economic Studies*, 49, 181–216.

Borio, C. (2011): “Rediscovering the macroeconomic roots of financial stability policy: journey, challenges, and a way forward,” *Annual Review of Finance*, Econ. 3,87–117.

Borio, C. E., and M. Drehmann (2009): “Assessing the risk of banking crises–revisited,” *BIS Quarterly Review,* Bank for international Settlements, March.

Borio, C., and P. Lowe (2002): “Asset prices, financial and monetary stability: exploring the nexus,” BIS Working Papers, No. 114.

Brunnermeier, M., A. Crocket, A. D. Persaud, and H. Shin (2009): “The fundamental principles of financial regulation,” Geneva Reports on the World Economy 11.

Bryant, R. C., P. Hooper, and C. Mann (eds.) (1993): *Evaluating Policy Regimes: New Research in Empirical Macroeconomics*. Brookings Institutions, Washington D.C.

Calvo, G. (1983): “Staggered Prices in a Utility-Maximizing Framework,” *Journal of Monetary Economics* 12 (3): 383-398.

Cairó, I., and J. Sim (2020): “Monetary Policy and Financial Stability,” Finance and Economics Discussion Series 2020-101. Washington: Board of Governors of the Federal Reserve System.

Caruana, J. (2010): “Macroprudential policy: working towards a new consensus,” *Remarks at the high-level meeting on “The Emerging Framework for Financial Regulation and Monetary Policy” Washington DC., USA, April 23, 2010*, retrieved February 16, 2021 from https://www.bis.org/speeches/sp100426.pdf.

Christiano, L., R. Motto, and R. Rostagno (2007): “Two Reasons Why Money and Credit May be Useful in Monetary Policy,” NBER Working Papers 13502, National Bureau of Economic Research.

Claessens, S., M. A. Kose, and M. E. Terrones (2009): “What happens during recessions, crunches and busts?,” *Economic Policy,* 24(60), 653-700.

Crockett, A. (2000): “Marrying the micro- and macro-prudential dimensions of financial stability,” *Speech given September 21, 2000 at the Eleventh International Conference of Banking Supervisors, Basel, Switzerland,* retrieved February 16, 2021 from https://www.bis.org/speeches/sp000921.htm.

Crotty, J. (2009): “Structural causes of the global Financial crisis: a critical assessment of the ‘new financial architecture’,” *Cambridge journal of economics,* 33 (4), 563-580.

Curdia, V., and M. Woodford (2010): “Credit spreads and monetary policy,” *Journal of Money, credit and Banking*, *42*, 3-35.

Debortoli, D., J. Kim, J. Lindé, and R. Nunes (2018): “Designing a Simple Loss Function for Central Banks: Does a Dual Mandate Make Sense?,” *The Economic Journal,* 129, 2010-2038.

Dell’Ariccia, G., Laeven, L., and R. Marquez, R. (2014): “Real interest rates, leverage, and bank risk-taking,” *Journal of Economic Theory*, 149, 65-99.

Dennis, R., and P. Ilbas (2016): "Monetary and Macroprudential Policy Games in a Monetary," Working Paper Research 304, National Bank of Belgium.

Evjen, S., and T. B. Kloster (2012): “Norges Bank's New Monetary Policy Loss Function- Further Discussion,” Norges Bank, Staff Memo No.11.

Filardo, A. J., and P. Rungcharoenkitkul (2016): “A quantitative case for leaning against the wind,” BIS Working Paper No 594.

Forlati, C., and L. Lambertini (2011): “Risky mortgages in a DSGE model,” *International Journal of Central Banking,* 7(1), 285-336.

FSB-BIS-IMF (2011): “Macroprudential Policy Tools and Frameworks: Update to g20 Finance Ministers and Central Bank Governors,” retrieved February 24, 2021 from https://www.bis.org/publ/othp17.htm.

Galati, G., and R. Moessner (2013): “Macroprudential policy–a literature review,” *Journal of Economic Surveys,* 27(5), 846-878.

Gelain, P., and P. Ilbas (2014): "Monetary and macroprudential policies in an estimated model with financial intermediation," Working Paper Research 258, National Bank of Belgium.

Gerali, A., S. Neri, L. Sessa, and F. M. Signoretti (2010): “Credit and Banking in a DSGE Model of the Euro Area,” *Journal of money, Credit and Banking*, 42, 107-141.

Gruen, D., M. Plumb, and A. Stone (2005): “How Should Monetary Policy Respond to Asset-Price Bubbles?,” *International Journal of Central Banking,* 1(3). 1-32.

Hannoun, H. (2010): “Towards a global financial stability framework,” *Speech given February 26-27, 2010 at the 45th SEACEN Governors’ Conference, Siem Reap Province, Cambodia,* retrieved February 16, 2021 from https://www.bis.org/speeches/sp100303.pdf.

He, Z., and A. Krishnamurthy (2019): “A macroeconomic framework for quantifying systemic risk,” *American Economic Journal: Macroeconomics*, *11*(4), 1-37.

Iacoviello, M. (2011): “Housing wealth and consumption,” FRB International Finance Discussion Paper, (1027).

IMF (2011): “Macroprudential policy: an organizing framework Background Paper 1,” Policy Papers 2011/018.

IMF (2015): “Monetary Policy and Financial Stability,” IMF Staff Report September 2015, IMF.

Jordà, Ò., M. Schularick, and A. M. Taylor (2013): “When credit bites back,” *Journal of Money, Credit and Banking*, *45*(s2), 3-28.

Kahou, M. E., and A. Lehar (2017): “Macroprudential policy: A review,” *Journal of financial stability*, 29, 92-105.

Kannan, P., P. Rabanal, and A. Scott (2012): “Monetary and Macroprudential Policy Rules in a Model with House Price Booms,” *The B.E. Journal of Macroeconomics,* 12(1), 1-44.

Kiley M. T. and J. W. Sim (2015): "Optimal Monetary and Macroprudential Policies: Gains and Pitfalls in a Model of Financial Intermediation," Finance and Economics Discussion Series 2015-78, Board of Governors of the Federal Reserve System (U.S.).

Macroeconomic Model Database (2021): “Online Comparison Platform,” retrieved January 30, 2021 from https://www.macromodelbase.com/download.

Medina, J. P., and J. Roldós (2018): “Monetary and macroprudential policies to manage capital flows,” *International Journal of Central Banking*, 52, 201-257.

Menz, J. O., and L. Vogel (2009): “A detailed derivation of the sticky price and sticky information new keynesian dsge model,” DEP (Socioeconomics) Discussion Papers-Macroeconomics and Finance Series (No. 2/2009).

Mishkin, F. (2008): “How Should We Respond to Asset Price Bubbles?,” *Speech given May 15, 2008 at the Wharton Financial Institutions Center and Oliver Wyman Institute’s Annual Financial Risk Roundtable, Philadelphia, PA.* retrieved February 16, 2021 from https://www.federalreserve.gov/newsevents/speech/mishkin20080515a.htm.

Mishkin, F. S. (2011): “Monetary policy strategy: lessons from the crisis,” NBER Working Papers 16755, National Bureau of Economic Research.

Nuguer, V., J. Roldan-Pena, E. Mendoza and J. Carrillo (2016): "When the Central Bank Meets the Financial Authority: Strategic Interactions and Institutional Design," 2016 Meeting Papers 1461, Society for Economic Dynamics.

Ozkan, M. F. G., and D. F. Unsal (2014): "On the use of Monetary and Macroprudential Policies for Small Open Economies," IMF Working Papers 2014/112, International Monetary Fund.

Paoli, B. D., and M. Paustian (2017): “Coordinating monetary and macroprudential policies,” *Journal of Money, Credit and Banking*, 49(2-3), 319-349.

Popoyan, L., M. Napoletano, and A. Roventini (2017): "Taming macroeconomic instability: Monetary and macro-prudential policy interactions in an agent-based model," Journal of Economic Behavior & Organization, Elsevier, vol. 134(C), pages 117-140.

Quint, D., and P. Rabanal (2014a): “Monetary and Macroprudential Policy in an Estimated DSGE Model of the Euro Area,” *International Journal of Central Banking,* 10(2), 169-236.

Quint, D., and P. Rabanal (2014b): “Appendix to Monetary and Macroprudential Policy in an Estimated DSGE Model of the Euro Area,” retrieved February 16, 2021 from http://www.paurabanal.net/QR_Appendix.pdf.

Rajan, R. G. (2006): “Has finance made the world riskier?,” *European financial management*, 12(4), 499-533.

Saunders, T., and P. Tulip (2019): “Cost-benefit Analysis of Leaning against the Wind,” RBA Research Discussion Papers RDP 2019-05, Reserve Bank of Australia.

Schinasi, G. J. (2005): *Safeguarding Financial Stability: Theory and Practice*. International Monetary Fund, Washington D.C.

Smets, F., and R. Wouters (2007): “Shocks and frictions in US business cycles: A Bayesian DSGE approach,” *American economic review,* 97(3), 586-606.

Svensson, L. E. O. (2017): "Cost–benefit analysis of leaning against the wind," *Journal of Monetary Economics,* 90, 193–213.

Taylor, J. B. (1993): "Discretion versus policy rules in practice," *Carnegie-Rochester conference series on public policy,* 39, 195-214.

Tinbergen, J. (1952): *On the Theory of Economic Policy*. 2nd edition, Amsterdam, North Holland.

Verona, F., M. M. F. Martins, and I. Drumond (2017): "Financial shocks, financial stability, and optimal Taylor rules," *Journal of Macroeconomics*, 54, 187-207.

Woodford, M. (2001): "The Taylor Rule and Optimal Monetary Policy," *The American Economic Review,* 91(2), 232-237.

Woodford, M. (2003): *Interest and Prices.* Princeton University Press, Princeton, NJ, and Oxford, UK.

Woodford, M. (2012): "Inflation targeting and financial stability," NBER Working Papers 17967, National Bureau of Economic Research.

# 7 Appendix

## 7.1 Model Derivation

This appendix provides a closer look at the model. It is based on Quint and Rabanal (2014a). Nevertheless, this section tries to present not only the model but also to fully derive its first-order conditions. Quint and Rabanal (2014b) state these conditions briefly but without deriving every component in detail.

The part dealing with the credit market and the financial accelerator is very similar to section *2.2* but it is replenished with further details regarding the derivation and the statistical properties. It is revised in the appendix to allow for an uninterrupted presentation of the model. First, the credit market and the financial frictions are introduced as they play an important role for household behaviour. Second, the two types of households are introduced, and their utility maximisation is presented. Third, the behaviour of intermediate and final good producers is explained.

### *7.1.1 The Credit Market and the Financial Accelerator*

The domestic and foreign credit markets include frictions which are modelled in the idea of the Financial Accelerator Model of BGG. As in BGG, the lending-deposit spread depends on the state of the housing market and there exists a default risk for borrowers. Nevertheless, there are important differences between the two approaches: first, there is no "costly state verification" or asymmetric information as in BGG. Consequently, borrowers will only default when they are really under water. Second, no collateral will be destroyed in the case of a default[28]. Third, the one-period domestic (foreign) lending rate is determined via the participation constraint of the risk-neutral domestic (foreign) financial intermediary.

*Domestic Financial Intermediaries*

Deposits from savers $S_t$ are taken by domestic financial intermediaries at a deposit rate $R_t$. These deposits are combined to loans $S_t^B$, granted to borrowers who must pay a lending rate $R_t^L$. The housing stock, with value $P_t^D D_t^B$, is placed as collateral, with $P_t^D$ representing the housing price and $D_t^B$ representing the housing stock of borrowers[29].

[28] If so, this could lead to unrealistic developments in the housing stock and in the investment decisions (see Forlati and Lambertini 2011).

[29] $S_t, S_t^B$ $and$ $D_t^B$ are denoted in per-capita quantities. As the maximisation problems of all savers and borrowers are symmetrical, the subscripts denoting the specific saver or borrower have been dropped. See section 7.1.2 for further reference.

Every borrower indexed by $j$ is hit by an idiosyncratic risk shock $\omega_t^j$ which affects the value of their housing stock. The shock is log-normally distributed, with $\log(\omega_t^j) \sim N\left(-\frac{\sigma_{\omega,t}^2}{2}, \sigma_{\omega,t}^2\right)$ and $\sigma_{\omega,t}$ being the standard deviation which depends on its one-period lagged value and a shock $\mu_{\omega,t}$:

$$\log(\sigma_{\omega,t}) = (1 - \rho_{\sigma_\omega})\log(\bar{\sigma}_\omega) + \rho_{\sigma_\omega}\log(\sigma_{\omega,t-1}) + \mu_{\omega,t}.$$

The cumulative distribution function is denoted $F(\omega; \sigma_{\omega,t})$ and $\mathrm{E}[\omega_t^j] = 1$. The actual realisation of $\omega_t^j$ is known at the end of the period and then it is common knowledge[30]. The household will only default if it is under water meaning that its outstanding debt is higher than the value of its housing stock. Formally, that is when $\omega_{t-1}^j P_t^D D_t^B$ is smaller than $R_{t-1}^L S_{t-1}^B$. Thus, the risk shock does directly affect the household's ability to repay its loans and, consequently, it affects the defaulting rate as well. If $\omega_{t-1}^j$ is high enough, the household will simply repay the full amount of its loan, that is $R_{t-1}^L S_{t-1}^B$. If the household has to default, it has to repay the still existing value of the housing stock, $\omega_{t-1}^j P_t^D D_t^B$, to a debt-collection agency. When this is done, the defaulted household can keep the house. The agency charges a fraction μ of this payment as a commission and transfers the rest to the domestic financial intermediary. As well as the financial intermediaries, debt-collection agencies are owned by savers who receive their profits at the end of the period.

The banks' expected return depends on the defaulting rate and, thus, on the realisation of $\omega_t^j$, which is only known in period *t+1*. Consequently, in period t banks only know the ex-ante threshold value of $\omega_t^j$ denoted $\bar{\omega}_t^a$ with

$$\bar{\omega}_t^a E_t[P_{t+1}^D D_{t+1}^B] = R_t^L S_t^B. \tag{10}$$

Knowing the real threshold $\bar{\omega}_t$, which indicates the boundary between repaying or defaulting, requires knowing future housing prices and the borrowers' future housing stocks.
Banks can use the ex-ante threshold to compute the expected fraction of borrowers who will default in the next period:

[30] Thus, no "costly state verification" is needed.

$$F(\overline{\omega}_t^{\alpha}, \sigma_{\omega,t}) = \int_0^{\overline{\omega}_t^{\alpha}} dF(\omega; \sigma_{\omega,t}) d\omega, \tag{11}$$

with $dF(\omega; \sigma_{\omega,t})$ being the probability distribution function. On the contrary, the fraction of borrowers who can repay their loans are given by

$$[1 - F(\overline{\omega}_t^{\alpha}, \sigma_{\omega,t})] = \int_{\overline{\omega}_t^{\alpha}}^{\infty} dF(\omega; \sigma_{\omega,t}) d\omega. \tag{12}$$

Conditional on defaulting, the expected value of the quality shock is characterised by

$$G(\overline{\omega}_t^{\alpha}, \sigma_{\omega,t}) = \int_{\overline{\omega}_t^{\alpha}}^{\infty} \omega F(\omega; \sigma_{\omega,t}) . \tag{13}$$

Thus, the expected repayment by defaulting borrowers is given by

$$G(\overline{\omega}_t^{\alpha}, \sigma_{\omega,t}) P_{t+1}^D D_{t+1}^B = \omega P_{t+1}^D D_{t+1}^B F(\omega; \sigma_{\omega,t}). \tag{14}$$

As the macroprudential intervention influences the banks' participation constraint, the channel used for that purpose is introduced now. The macroprudential policy tool is designed to affect the domestic and foreign financial intermediaries' balance sheets. This modelling approach is similar to Kannan et al. (2012). The balance sheet of a domestic bank (as well as for a foreign bank) is given by:

$$n\,\lambda \frac{1}{\eta_t}(S_t - B_t) = n(1-\lambda) S_t^B, \tag{15}$$

whereby $n$ is the size of the domestic country ($1-n$ is the foreign country's size) and λ is the mass of savers ($1-\lambda$ is the mass of borrowers) – with λ having the same value in both countries. $S_t, B_t\ and\ S_t^B$ have to be multiplied by the country's size and the masses as they are in per-capita terms and the balance sheet consists of the aggregate amounts. The macroprudential policy instrument is denoted $\eta_t$. Changing this instrument limits or broadens the amount of loans the banks can lend. A tightening of macroprudential policy could be realised through a higher value of $\eta_t$. This would reduce the amount of loans banks can lend.

Financial intermediaries are risk neutral. This assumption implies that they require the expected return from the granted amount of credit to be equal to the deposit rate, $R_t$, that banks owe the savers. This is the domestic banks' participation constraint. Formally, using equations (14), it is

$$n\lambda R_t(S_t - B_t)$$
$$= n(1-\lambda)E_t\{(1-\mu)G(\overline{\omega}_t^\alpha, \sigma_{\omega,t})P_{t+1}^D D_{t+1}^B + [1 - F(\overline{\omega}_t^\alpha, \sigma_{\omega,t})]\, R_t^L S_t^B\}. \tag{16}$$

Using (15) the constraint can be rewritten in order to clarify the macroprudential policy channel. First, solve (15) for $\lambda(S_t - B_t)$. Inserting the result in (16), simplifying and dividing through $S_t^B$leads to (17)

$$n\eta_t R_t(1-\lambda)S_t^B$$
$$= n(1-\lambda)E_t\{(1-\mu)G(\overline{\omega}_t^\alpha, \sigma_{\omega,t})P_{t+1}^D D_{t+1}^B + [1 - F(\overline{\omega}_t^\alpha, \sigma_{\omega,t})]\, R_t^L S_t^B\}$$
$$\Leftrightarrow \eta_t R_t = E_t\left\{(1-\mu)G(\overline{\omega}_t^\alpha, \sigma_{\omega,t})\frac{P_{t+1}^D D_{t+1}^B}{S_t^B} + [1 - F(\overline{\omega}_t^\alpha, \sigma_{\omega,t})]\, R_t^L\right\}. \tag{17}$$

In expectations, (17) must be fulfilled and banks set the lending rate and the credit amount appropriately. Moreover, (17) implies that domestic financial intermediaries do not make profits ex-ante. Nevertheless, domestic banks can make profits or losses ex-post. Profits will be transferred to savers who own the banks and who would recapitalise the banks in case of losses. One can use equation (10) and (17) to derive the lending deposit spread:

$$\frac{R_t^L}{R_t} = E_t\left\{\frac{\eta_t}{\frac{(1-\mu)G(\overline{\omega}_t^\alpha, \sigma_{\omega,t})}{\overline{\omega}_t^\alpha} + [1 - F(\overline{\omega}_t^\alpha, \sigma_{\omega,t})]}\right\}. \tag{18}$$

Equation (9) shows that the lending deposit spread depends positively on the macroprudential policy tool. A higher value of $\eta_t$ leads to a higher lending rate. An increase in the interest rate has the same effect.

*Foreign Financial Intermediaries*

The foreign credit market works in the same manner as the domestic market, with foreign financial intermediaries being the equivalent of domestic financial intermediaries. Consequently, the foreign lending rate $R_t^{L*}$ is set analogously to the domestic lending rate. The domestic deposit rate is assumed to equal the risk-free rate set by the central bank. The foreign deposit rate $R_t^*$, however, does not equal the risk-free rate. It is set by international intermediaries at the international bond market, which is explained in the next section.

*International Financial Intermediaries*

When domestic banks have surplus funds $B_t$ (also in per-capita quantities), they sell them to international intermediaries that will lend them to foreign banks at a rate of $R_t^*$. They set $R_t^*$ to be equal to the domestic deposit rate plus a risk premium. The spread depends on the ratio of real net foreign assets to the steady-state value of the domestic non-durable GDP. International intermediaries use the following formula to set $R_t^*$:

$$R_t^* = R_t + \left\{\vartheta_t \, exp\left[\kappa_B\left(\frac{B_t}{P_t^C Y^C}\right)\right] - 1\right\}, \qquad (19)$$

with $B_t/P_t^C$ being real foreign assets and $Y^C$ being the steady-state value of non-durable domestic GDP. The elasticity of the risk premium is denoted $\kappa_B$, $\vartheta_t$ denotes an exogenous shock to the risk premium which can increase the spread between domestic and foreign deposit rates. When the domestic banks sell bonds on the international market, which means $B_t$ is higher than zero, foreign banks must pay a higher deposit rate than the domestic banks.

International intermediaries make positive profits equal to $(R_t^* - R_t)B_t$. They are owned by savers from both countries and each saver receives an equal share of the profits.

### *7.1.2 Households*

In both countries, the share of savers is equal to $\lambda$ and the share of borrowers is equal to *1-λ*. The two types of agents differ in their habit formation preference and their discount factor. Moreover, savers receive profits from financial intermediaries and debt collection agencies as well as from intermediate goods producers.

*Savers*

A representative saver of the domestic country j with $j \in [0, \lambda]$ tries to maximise its utility function:

$$E_0\left\{\sum_{t=0}^{\infty} \beta^t \left[\gamma\, \xi_t^C \, log\left(C_t^j - \varepsilon C_{t-1}\right) + (1-\gamma)\xi_t^D \, log\left(D_t^j\right) - \frac{\left(L_t^j\right)^{1+\varphi}}{1+\varphi}\right]\right\}. \qquad (20)$$

$C_t^j$ represents saver j's consumption of non-durable goods, $D_t^j$ the stock of durables (housing stock) and $L_t^j$ the labour disutility. The household's habit formation parameter ε measures the influence of the aggregate consumption of period *t-1*, $C_{t-1}$. The weights of durables and non-durables are given by the parameter γ and 1-γ, respectively. The parameter φ denotes the inverse elasticity of labour supply and β represents the discount factor of all savers. Two preference shocks hit the saver's utility: $\xi_t^C$ affects the marginal utility of non-durable consumption, $\xi_t^D$ the marginal utility of the housing stock.

The household can consume both domestic non-durables and foreign imported non-durables. Thus, $C_t^j$ is an index of the domestic ($C_{H,t}^j$) and the foreign ($C_{F,t}^j$) consumption:

$$C_t^j = \left[\tau^{\frac{1}{\iota_C}} \left(C_{H,t}^j\right)^{\frac{\iota_C - 1}{\iota_C}} + (1-\tau)^{\frac{1}{\tau_C}} \left(C_{F,t}^j\right)^{\frac{\iota_C - 1}{\iota_C}}\right]^{\frac{\iota_C}{\iota_C - 1}}, \tag{21}$$

where the household's preference for domestic consumption is measured by $\tau \in [0,1]$ and $\iota_C$ is the elasticity of substitution between the two types of goods. For $\iota_C \to \infty$, they become perfect substitutes as the weights become equal: $\lim_{\iota_C \to \infty} \tau^{\frac{1}{\iota_C}} = \lim_{\iota_C \to \infty} (1-\tau)^{\frac{1}{\tau_C}} = 1$ because of $\tau, 1-\tau > 0$. Furthermore, the following relationship must hold:

$$P_t^C C_t^j = P_t^H C_{H,t}^j + P_t^F C_{F,t}^j, \tag{22}$$

where $P_t^H$ and $P_t^F$ denote the price of domestic and foreign non-durables and $P_t^C$ is the aggregate price index.

Saver j provides labour to the non-durables and to the durables production sector. Formally, this relationship is modelled similarly to the consumption index:

$$L_t^j = \left[\alpha^{-\iota_L} \left(L_t^{C,j}\right)^{1+\iota_L} + (1-\alpha)^{-\iota_L} \left(L_t^{D,j}\right)^{1+\iota_L}\right]^{\frac{1}{1+\iota_L}}. \tag{23}$$

Labour disutility consists of disutility from work in the non-durable sector, $L_t^{C,j}$, and from work in the durable sector, $L_t^{D,j}$. The preference for work in either of the two sectors is measured by α. In the steady state, the share of total labour used in the non-durable sector is equal to α. The parameter $\iota_L$ determines the substitutability of the two types of labour and in the context of work it can be regarded as the cost of reallocating labour between the sectors. Wages in the sectors are flexible and equivalent to the marginal rate of substitution between consumption and labour in every sector. For values of $\iota_L$ higher than zero, the wages differ.

Household j must take the following budget constraint into account:

$$P_t^C C_t^j + P_t^D I_t^j + S_t^j \leq R_{t-1} S_{t-1}^j + W_t^C L_t^{C,j} + W_t^D L_t^{D,j} + \Pi_t^j, \tag{24}$$

with $P_t^C$ and $P_t^D$ representing the price of non-durables and durables. It can buy non-durables, $C_t^j$, invest in the stock of housing, $I_t^j$, or it has access to deposits, $S_t^j$. $R_t$ marks the deposit interest rate and $\Pi_t^j$ incicates the received profits. The wage in the non-durable sector is $W_t^C$, and $W_t^D$ is the wage received from work in the durable sector. Residential investment, $I_t^j$, and the stock of housing, $D_t^j$, interrelate in the following law of motion:

$$D_t^j = (1-\delta) D_{t-1}^j + \left[1 - F\left(\frac{I_{t-1}^j}{I_{t-2}^j}\right)\right] I_{t-1}^j \,. \tag{25}$$

The share of the housing stock that depreciates is denoted by δ and F(·) represents adjustment costs. Quint and Rabanal (2014a) implement this convex function to model hump-shaped responses of investment when hit by a shock and to reduce the investment volatility. Besides, F(·) is equal to zero in the steady state as well as the first derivative. The second derivative is greater than zero in steady state. Formally, $\overline{F} = \overline{F'} = 0$ and $\overline{F''} > 0$ must hold with the bar representing steady state values.

The representative household j wants to maximise (20) under the conditions of (21) – (25). In the original paper, the maximisation problem is split into two stages: first, the household decides how much it wants to spend on non-durable and durable consumption as well as how it wants to allocate its labour. Second, it chooses the share of domestic and foreign non-durable consumption.

Referring to the first step, the household wants to maximise (20) subject to (23), (24) and (25). For simplicity, (23) is plugged in into (20). The following Lagrange function represents the problem[31]:

$$\max_{C_t, D_t, I_t, S_t, L_t^C, L_t^D} \mathcal{L} =$$

$$E_0 \sum_{t=0}^{\infty} \beta^t \{[\gamma\, \xi_t^C \, log(C_t - \varepsilon C_{t-1}) + (1-\gamma) \xi_t^D \, log(D_t)$$

[31] Since all savers are identical and face the same problem, the subscript j can be dropped.

$$-\frac{\left[\alpha^{-\iota_L}\left(L_t^C\right)^{1+\iota_L}+(1-\alpha)^{-\iota_L}\left(L_t^D\right)^{1+\iota_L}\right]^{\frac{1+\varphi}{1+\iota_L}}}{1+\varphi}\Bigg]$$

$$+\lambda_t^{BC}[R_{t-1}S_{t-1}+W_t^C L_t^C+W_t^D L_t^D+\Pi_t-P_t^C C_t-P_t^D I_t-S_t]$$

$$+\lambda_t^{LM}\left[(1-\delta)D_t+\left[1-F\left(\frac{I_{t-1}}{I_{t-2}}\right)\right]I_{t-1}-D_t\right]\Big\} \quad (26)$$

with $\lambda_t^{BC}$ and $\lambda_t^{LM}$ being the Lagrangian-multiplier regarding the budget constraint and the law of motion. The corresponding first order conditions are[32]:

$$\frac{\partial\mathcal{L}}{\partial C_t}=\beta^t\left(\frac{\gamma\,\xi_t^C}{C_t-\varepsilon C_{t-1}}-\lambda_t^{BC}P_t^C\right)=0, \quad (27)$$

$$\frac{\partial\mathcal{L}}{\partial D_t}=\beta^t\left((1-\gamma)\,\frac{\xi_t^D}{D_t}-\lambda_t^{LM}\right)+\beta^{t+1}E_t[\lambda_{t+1}^{LM}](1-\delta)=0 \quad (28)$$

$$\frac{\partial\mathcal{L}}{\partial I_t}=-\beta^t\lambda_t^{BC}P_t^D+E_t\left[\beta^{t+1}\left(\lambda_{t+1}^{LM}\left(1-F\left(\frac{I_t}{I_{t-1}}\right)-F'\left(\frac{I_t}{I_{t-1}}\right)\frac{I_t}{I_{t-1}}\right)\right)\right.$$

$$\left.+\beta^{t+2}\lambda_{t+2}^{LM}\left(F'\left(\frac{I_{t+1}}{I_t}\right)\left(\frac{I_{t+1}}{I_t}\right)^2\right)\right]=0, \quad (29)$$

$$\frac{\partial\mathcal{L}}{\partial S_t}=-\beta^t\lambda_t^{BC}+\beta^{t+1}\lambda_{t+1}^{BC}R_t=0, \quad (30)$$

$$\frac{\partial\mathcal{L}}{\partial L_t^C}=\beta^t\left[-\alpha^{-\iota_L}\,(L_t^C)^{\iota_L}(\alpha^{-\iota_L}(L_t^C)^{1+\iota_L}+(1-\alpha)^{-\iota_L}(L_t^D)^{1+\iota_L})^{\frac{\varphi-\iota_L}{1-\iota_L}}+\lambda_t^{BC}W_t^C\right]=0, \quad (31)$$

$$\frac{\partial\mathcal{L}}{\partial L_t^D}=\beta^t\left[-(1-\alpha)^{-\iota_L}\,(L_t^D)^{\iota_L}(\alpha^{-\iota_L}(L_t^C)^{1+\iota_L}+(1-\alpha)^{-\iota_L}(L_t^D)^{1+\iota_L})^{\frac{\varphi-\iota_L}{1-\iota_L}}+\lambda_t^{BC}W_t^D\right]=0, \quad (32)$$

For deriving the Euler-equation, (27) and (30) are solved for $\lambda_t^{BC}$. It follows:

$$\lambda_t^{BC}=\frac{\gamma\,\xi_t^C}{(C_t-\varepsilon C_{t-1})P_t^C}, \quad (27')$$

and $$\lambda_t^{BC}=\beta\lambda_{t+1}^{BC}R_t. \quad (30')$$

Moreover, (27') implies

$$E_t\lambda_{t+1}^{BC}=E_t\frac{\gamma\,\xi_{t+1}^C}{(C_{t+1}-\varepsilon C_t)P_{t+1}^C}. \quad (27'')$$

Plugging (27') and (27'') into (30') and dividing by (27') leads to the model's Euler-equation:

[32] The derivatives with respect to $\lambda_t^{BC}$ and $\lambda_t^{LM}$ are skipped to keep the derivation as parsimonious as possible.

$$1 = \beta R_t E_t\left[\frac{P_t^C}{P_{t+1}^C}\frac{\xi_{t+1}^C}{\xi_t^C}\frac{C_t-\varepsilon C_{t-1}}{C_{t+1}-\varepsilon C_t}\right]. \quad (33)$$

The demand for durable goods follows directly from (28):

$$(1-\gamma)\,\frac{\xi_t^D}{D_t} = \lambda_t^{LM} + \beta(1-\delta)E_t[\lambda_{t+1}^{LM}] = 0. \quad (34)$$

The investment decision can be derived by inserting (27') into (29) and solving for $\frac{\gamma\,\xi_t^C}{(C_t-\varepsilon C_{t-1})}\frac{P_t^D}{P_t^C}$:

$$\frac{\gamma\,\xi_t^C}{(C_t-\varepsilon C_{t-1})}\frac{P_t^D}{P_t^C} = E_t\left[\beta\left(\lambda_{t+1}^{LM}\left(1-F\left(\frac{I_t}{I_{t-1}}\right)-F'\left(\frac{I_t}{I_{t-1}}\right)\frac{I_t}{I_{t-1}}\right)\right).\right.$$
$$\left.+\beta^2\lambda_{t+2}^{LM}\left(F'\left(\frac{I_{t+1}}{I_t}\right)\left(\frac{I_{t+1}}{I_t}\right)^2\right)\right] \quad (35)$$

Taking equation (13) to the power of $\varphi-\iota_L$, it follows $L_t^{\varphi-\iota_L} = [\alpha^{-\iota_L}(L_t^C)^{1+\iota_L} + (1-\alpha)^{-\iota_L}(L_t^D)^{1+\iota_L}]^{\frac{\varphi-\iota_L}{1+\iota_L}}$. This result can be used to simplify (31) and (32):

$$\beta^t[-\alpha^{-\iota_L}\,(L_t^C)^{\iota_L}{L_t}^{\varphi-\iota_L} + \lambda_t^{BC}W_t^C] = 0, \quad (31')$$
$$\beta^t[-(1-\alpha)^{-\iota_L}\,(L_t^D)^{\iota_L}{L_t}^{\varphi-\iota_L} + \lambda_t^{BC}W_t^D] = 0, \quad (32')$$

Finally, the labour supply decisions are obtained by plugging (27') into (31') and (32'), respectively, and then solving for $\frac{\gamma\,\xi_t^C W_t^C}{(C_t-\varepsilon C_{t-1})}$ or $\frac{\gamma\,\xi_t^C W_t^D}{(C_t-\varepsilon C_{t-1})}$ leads to[33]:

$$-\alpha^{-\iota_L}\,(L_t^C)^{\iota_L}{L_t}^{\varphi-\iota_L} = \frac{\gamma\,\xi_t^C W_t^C}{(C_t-\varepsilon C_{t-1})P_t^C}, \quad (36)$$
$$-(1-\alpha)^{-\iota_L}\,(L_t^D)^{\iota_L}{L_t}^{\varphi-\iota_L} = \frac{\gamma\,\xi_t^C W_t^D}{(C_t-\varepsilon C_{t-1})P_t^C}, \quad (37)$$

What remains is the second part of the saver's maximisation.

The saver wants to minimise its cost for a given level of non-durable consumption, $Z_t$ by choosing the optimal level of domestic and foreign non-durable consumption. Formally stated, the problem is

[33] In their model description, Quint and Rabanal (2014b) do not include $P_t^C$ in the denominator of equation (36) and (37). However, my derivation is in line with the results of Kannan et al. (2012, p.6).

$$\min_{C_{H,t},C_{F,t}} C_{H,t}P_t^H + C_{F,t}P_t^F \qquad \text{s.t.} \left[\tau^{\frac{1}{\iota_C}}\left(C_{H,t}\right)^{\frac{\iota_C-1}{\iota_C}} + (1-\tau)^{\frac{1}{\tau_C}}\left(C_{F,t}\right)^{\frac{\iota_C-1}{\iota_C}}\right]^{\frac{\iota_C}{\iota_C-1}} = C_t\,,$$

which leads to a Lagrange-function of the form

$$\min_{C_{H,t},C_{F,t}} \mathcal{L} = C_{H,t}P_t^H + C_{F,t}P_t^F$$
$$+\lambda_t^{DFP}\left(C_t - \left[\tau^{\frac{1}{\iota_C}}\left(C_{H,t}\right)^{\frac{\iota_C-1}{\iota_C}} + (1-\tau)^{\frac{1}{\tau_C}}\left(C_{F,t}\right)^{\frac{\iota_C-1}{\iota_C}}\right]^{\frac{\iota_C}{\iota_C-1}}\right). \qquad (38)$$

The first-order conditions are

$$\frac{\partial\mathcal{L}}{\partial C_{H,t}} = P_t^H - \lambda_t^{DFP}\tau^{\frac{1}{\iota_C}}\left(C_{H,t}\right)^{\frac{-1}{\iota_C}}\left[\tau^{\frac{1}{\iota_C}}\left(C_{H,t}\right)^{\frac{\iota_C-1}{\iota_C}} + (1-\tau)^{\frac{1}{\tau_C}}\left(C_{F,t}\right)^{\frac{\iota_C-1}{\iota_C}}\right]^{\frac{1}{\iota_C-1}} = 0, \qquad (39)$$

$$\frac{\partial\mathcal{L}}{\partial C_{F,t}} = P_t^F - \lambda_t^{DFP}(1-\tau)^{\frac{1}{\tau_C}}\left(C_{F,t}\right)^{\frac{-1}{\iota_C}}\left[\tau^{\frac{1}{\iota_C}}\left(C_{H,t}\right)^{\frac{\iota_C-1}{\iota_C}} + (1-\tau)^{\frac{1}{\tau_C}}\left(C_{F,t}\right)^{\frac{\iota_C-1}{\iota_C}}\right]^{\frac{1}{\iota_C-1}} = 0. \qquad (40)$$

Simplifying (39) and (40) with (21) and solving for $C_{H,t}$ and $C_{F,t}$ leads to

$$C_{H,t} = \tau\left(\frac{P_t^H}{\lambda_t^{DFP}}\right)^{-\iota_C} C_t, \qquad (39')$$

and $$C_{F,t} = (1-\tau)\left(\frac{P_t^F}{\lambda_t^{DFP}}\right)^{-\iota_C} C_t. \qquad (40')$$

Next, (39') and (40') are plugged in into equation (21) and solved for $\lambda_t^{DFP}$:

$$C_t = \left[\tau^{\frac{1}{\iota_C}}\left(\tau\left(\frac{P_t^H}{\lambda_t^{DFP}}\right)^{-\iota_C} C_t\right)^{\frac{\iota_C-1}{\iota_C}} + (1-\tau)^{\frac{1}{\tau_C}}\left((1-\tau)\left(\frac{P_t^F}{\lambda_t^{DFP}}\right)^{-\iota_C} C_t\right)^{\frac{\iota_C-1}{\iota_C}}\right]^{\frac{\iota_C}{\iota_C-1}},$$

$$\Leftrightarrow C_t = \left[\tau^{\frac{1}{\iota_C}}\tau^{\frac{\iota_C-1}{\iota_C}}\left(\frac{P_t^H}{\lambda_t^{DFP}}\right)^{1-\iota_C}(C_t)^{\frac{\iota_C-1}{\iota_C}} + (1-\tau)^{\frac{1}{\tau_C}}(1-\tau)^{\frac{\iota_C-1}{\tau_C}}\left(\frac{P_t^F}{\lambda_t^{DFP}}\right)^{1-\iota_C}(C_t)^{\frac{\iota_C-1}{\iota_C}}\right]^{\frac{\iota_C}{\iota_C-1}},$$

$$\Leftrightarrow C_t = C_t(\lambda_t^{DFP})^{\iota_C}\left[\tau\left(P_t^H\right)^{1-\iota_C} + (1-\tau)P_{F,t}{}^{1-\iota_C}\right]^{\frac{\iota_C}{\iota_C-1}},$$

$$\Leftrightarrow (\lambda_t^{DFP})^{-\iota_C} = \left[\tau\left(P_t^H\right)^{1-\iota_C} + (1-\tau)\left(P_t^F\right)^{1-\iota_C}\right]^{\frac{\iota_C}{\iota_C-1}},$$

$$\Leftrightarrow \left(\lambda_t^{DFP}\right)^{1-\iota_C} = \left[\tau\left(P_t^H\right)^{1-\iota_C} + (1-\tau)\left(P_t^F\right)^{1-\iota_C}\right]. \quad (41)$$

The aggregate price index, $P_t^C$, is defined through equation (22) as follows. Following Menz and Vogel (2009), $P_t^C$ is equal to $\lambda_t^{DFP}$. This equality can simply be shown by plugging in (41), (39') and (40') into (22).

Inserting the aggregate price index into the demand functions of domestic, (39'), and foreign non-durables, (40'), they become

$$C_{H,t} = \tau\left(\frac{P_t^H}{P_t^C}\right)^{-\iota_C} C_t, \quad (42)$$

$$C_{F,t} = (1-\tau)\left(\frac{P_t^F}{P_t^C}\right)^{-\iota_C} C_t. \quad (43)$$

*Borrowers*

Like savers, borrowers consume non-durables and durables and provide labour to both sectors. The wages of savers and borrowers are the same since firms cannot discriminate between the two types.

Contrary to savers, borrowers are more impatient and have a higher preference for early consumption that means $\beta^B < \beta$, where the subscript *B* indicates borrowers. The two levels of impatience are the reason why savers are willing to postpone their consumption by accumulating deposits and borrowers are taking loans which are covered by their housing wealth. Moreover, they have a different habit formation parameter $\varepsilon^B$ and they do not earn any profits made by financial institutions or firms. As discussed in section *6.1.1,* their housing stock is affected by an idiosyncratic risk shock $\omega_t^j$.

The utility function of a representative borrower $j \in [\lambda, 1]$ is equivalent to the one of savers:

$$E_0\left\{\sum_{t=0}^{\infty}(\beta^B)^t\left[\gamma\, \xi_t^C \, log\left(C_t^{B,j} - \varepsilon^B C_{t-1}^B\right) + (1-\gamma)\xi_t^D \, log\left(D_t^{B,j}\right) - \frac{\left(L_t^{B,j}\right)^{1+\varphi}}{1+\varphi}\right]\right\}. \quad (44)$$

Variables with the subscript *B* are specific to borrowers but the meaning of the variables remains the same. Borrowers face the same consumption (45) and labour indices (46) as well as the same law of motion for the housing stock (47):

$$C_t^{B,j} = \left[\tau^{\frac{1}{\iota_C}}\left(C_{H,t}^{B,j}\right)^{\frac{\iota_C-1}{\iota_C}} + (1-\tau)^{\frac{1}{\tau_C}}\left(C_{F,t}^{B,j}\right)^{\frac{\iota_C-1}{\iota_C}}\right]^{\frac{\iota_C}{\iota_C-1}}, \tag{45}$$

$$L_t^{B,j} = \left[\alpha^{-\iota_L}\left(L_t^{C,B,j}\right)^{1+\iota_L} + (1-\alpha)^{-\iota_L}\left(L_t^{D,B,j}\right)^{1+\iota_L}\right]^{\frac{1}{1+\iota_L}}, \tag{46}$$

$$D_t^{B,j} = (1-\delta)D_{t-1}^{B,j} + \left[1 - F\left(\frac{I_{t-1}^{B,j}}{I_{t-2}^{B,j}}\right)\right] I_{t-1}^{B,j}. \tag{47}$$

Since the amount of expenditures can differ among borrowers as they can default, the budget constraint does also differ. The constraint faced by borrowers who repay their loans completely is given by

$$P_t^C C_t^{B,j} + P_t^D I_t^{B,j} + R_{t-1}^L S_{t-1}^{B,j} \leq S_t^{B,j} + W_t^C L_t^{C,B,j} + W_t^D L_t^{D,B,j}, \tag{48}$$

and the constraint of defaulting borrowers is

$$P_t^C C_t^{B,j} + P_t^D I_t^{B,j} + \omega_{t-1}^j P_t^D D_t^{B,j} \leq S_t^{B,j} + W_t^C L_t^{C,B,j} + W_t^D L_t^{D,B,j}. \tag{49}$$

As mentioned above, the realisation of $\omega_{t-1}^j$ determines whether borrowers have to default or not. The ex-post critical value of the shock[34], $\bar{\omega}_{t-1}^p$, is defined as

$$\bar{\omega}_{t-1}^p P_t^D D_t^B = R_{t-1}^L S_{t-1}^B. \tag{50}$$

The ex-post threshold and the ex-ante threshold are the same in expectations when the loan contract is signed. Using the ex-post threshold, one can calculate the fraction of borrowers that defaults:

$$F\left(\bar{\omega}_{t-1}^p, \sigma_{\omega,t-1}\right) = \int_0^{\bar{\omega}_t^p} dF\left(\omega; \sigma_{\omega,t-1}\right) d\omega, \tag{51}$$

and consequently, the portion of borrowers that fully repays their loan, as their realisation of $\omega_{t-1}^j$ is high enough, is

[34] Since the maximisation problem is symmetric, the threshold has the same value for all borrowers. Thus, the subscripts can be dropped.

$$\left[1-F\left(\overline{\omega}_{t-1}^{p},\sigma_{\omega,t-1}\right)\right]=\int_{\overline{\omega}_{t}^{p}}^{\infty}dF\left(\omega;\sigma_{\omega,t-1}\right)d\omega. \quad (52)$$

To define the mean payment of defaulting borrowers to debt-collection agencies, the mean realisation of the shock conditional on being smaller than $\overline{\omega}_{t-1}^{p}$ is given by

$$G\left(\overline{\omega}_{t-1}^{p},\sigma_{\omega,t}\right)=\int_{\overline{\omega}_{t-1}^{p}}^{\infty}\omega F\left(\omega;\sigma_{\omega,t-1}\right). \quad (53)$$

Therefore, the mean payment to debt-collection agencies is

$$P_{t}^{D}G\left(\overline{\omega}_{t-1}^{p},\sigma_{\omega,t}\right)D_{t}^{B}=P_{t}^{D}\int_{\overline{\omega}_{t-1}^{p}}^{\infty}\omega F\left(\omega;\sigma_{\omega,t-1}\right)D_{t}^{B}. \quad (54)$$

With the given information of equations (50) to (54), it is possible to aggregate the two initial budget constraints (48) and (49). For simplicity, the j subscript can be dropped now:

$$P_{t}^{C}C_{t}^{B}+P_{t}^{D}\left[I_{t}^{B}+G\left(\overline{\omega}_{t-1}^{p},\sigma_{\omega,t}\right)D_{t}^{B}\right]+\left[1-F\left(\overline{\omega}_{t-1}^{p},\sigma_{\omega,t-1}\right)\right]R_{t-1}^{L}S_{t-1}^{B}$$
$$\leq S_{t}^{B}+W_{t}^{C}L_{t}^{C,B}+W_{t}^{D}L_{t}^{D,B}\;. \quad (55)$$

To further simplify the budget constraint, the ex-post rate of return, $R_{t}^{D}$[35], which banks receive from defaulted borrowers[36] is given by

$$R_{t}^{D}=\frac{P_{t}^{D}G\left(\overline{\omega}_{t-1}^{p},\sigma_{\omega,t}\right)D_{t}^{B}}{S_{t-1}^{B}}. \quad (56)$$

Plugging (56) into (55) yields to

$$P_{t}^{C}C_{t}^{B}+P_{t}^{D}I_{t}^{B}+\left\{R_{t}^{D}+\left[1-F\left(\overline{\omega}_{t-1}^{p},\sigma_{\omega,t-1}\right)\right]R_{t-1}^{L}\right\}S_{t-1}^{B}$$
$$\leq S_{t}^{B}+W_{t}^{C}L_{t}^{C,B}+W_{t}^{D}L_{t}^{D,B}. \quad (57)$$

[35] As this rate of return is only known in period t, the variables timing is consistent with $R_{t-1}^{L}$ which has been known one period before.
[36] Note that this rate of return does not include the fees μ charged by debt-collection agencies.

The utility maximisation of borrowers implies maximising (44) subject to (22), (45) to (47) and (57). Again, the labour index is plugged into the utility function. Formally stated, borrowers face the following Langrange function which is very similar to the problem faced by savers:

$$\max_{C_t^B, D_t^B, I_t^B, S_t^B, L_t^{C,B}, L_t^{D,B}} \mathcal{L} =$$

$$E_0 \sum_{t=0}^{\infty} (\beta^B)^t \{[\gamma\, \xi_t^C\, log(C_t^B - \varepsilon^B C_{t-1}^B) + (1-\gamma)\xi_t^D\, log(D_t^B)$$

$$- \frac{\left[\alpha^{-\iota_L}\left(L_t^{C,B}\right)^{1+\iota_L} + (1-\alpha)^{-\iota_L}\left(L_t^{D,B}\right)^{1+\iota_L}\right]^{\frac{1+\varphi}{1+\iota_L}}}{1+\varphi}\Bigg]$$

$$+\lambda_t^{BCB}\big[S_t^B + W_t^C L_t^{C,B} + W_t^D L_t^{D,B} - P_t^C C_t^B - P_t^D I_t^B$$

$$-\{R_t^D + [1 - F(\bar{\omega}_{t-1}^p, \sigma_{\omega,t-1})]R_{t-1}^L\}S_{t-1}^B\big]$$

$$+\lambda_t^{LM}\ \left[(1-\delta)D_{t-1}^B + \left[1 - F\left(\frac{I_{t-1}^B}{I_{t-2}^B}\right)\right] I_{t-1}^B\right] - D_t^B\Big\} \tag{58}$$

Technically, the maximisation problem is solved equivalently to the one faced by savers. Thus, a detailed derivation is relinquished, and I refer to the discussion of the savers' utility maximisation.

The borrowers' Euler equation (59), the demand for durables (60) and the demand for residential investment (61) have the following forms:

$$1 = \beta E_t \left\{\{[1 - F(\bar{\omega}_t^p, \sigma_{\omega,t})]R_t^L + R_{t+1}^D\}\left[\frac{P_t^C}{P_{t+1}^C}\frac{\xi_{t+1}^C}{\xi_t^C}\left(\frac{C_t^B - \varepsilon^B C_{t-1}^B}{C_{t+1}^B - \varepsilon^B C_t^B}\right)\right]\right\}, \tag{59}$$

$$(1-\gamma)\ \frac{\xi_t^D}{D_t^B} = \lambda_t^{LMB} + \beta^B(1-\delta)E_t[\lambda_{t+1}^{LMB}] = 0, \tag{60}$$

$$\frac{\gamma\, \xi_t^C}{(C_t^B - \varepsilon^B C_{t-1}^B)}\frac{P_t^D}{P_t^C} = E_t\left[\beta^B\left(\lambda_{t+1}^{LMB}\left(1 - F\left(\frac{I_t^B}{I_{t-1}^B}\right) - F'\left(\frac{I_t^B}{I_{t-1}^B}\right)\frac{I_t^B}{I_{t-1}^B}\right)\right)\right.$$

$$\left. +\beta^2 \lambda_{t+2}^{LM}\left(F'\left(\frac{I_{t+1}^B}{I_t^B}\right)\left(\frac{I_{t+1}^B}{I_t^B}\right)^2\right)\right]. \tag{61}$$

Borrowers split their labour according to[37]

[37] The same argument applies as in footnote 33.

$$-\alpha^{-\iota_L}\,(L_t^{C,B})^{\iota_L}(L_t^B)^{\varphi-\iota_L}=\frac{\gamma\,\xi_t^C W_t^C}{(C_t^B-\varepsilon^B C_{t-1}^B)P_t^C},\tag{62}$$

$$-(1-\alpha)^{-\iota_L}\,(L_t^{D,B})^{\iota_L}(L_t^B)^{\varphi-\iota_L}=\frac{\gamma\,\xi_t^C W_t^D}{(C_t^B-\varepsilon^B C_{t-1}^B)P_t^C}.\tag{63}$$

Borrowers' non-durable consumption is an index composed of domestic and foreign non-durables. It is analogously defined to the savers' non-durable index:

$$C_t^j=\left[\tau^{\frac{1}{\iota_C}}\left(C_{H,t}^j\right)^{\frac{\iota_C-1}{\iota_C}}+(1-\tau)^{\frac{1}{\tau_C}}\left(C_{F,t}^j\right)^{\frac{\iota_C-1}{\iota_C}}\right]^{\frac{\iota_C}{\iota_C-1}}.\tag{64}$$

Cost minimisation implies the following demand functions for domestic and foreign non-durables:

$$C_{H,t}^B=\tau\left(\frac{P_t^H}{P_t^C}\right)^{-\iota_C}C_t^B,\tag{65}$$

$$C_{F,t}^B=(1-\tau)\left(\frac{P_t^F}{P_t^C}\right)^{-\iota_C}C_t^B.\tag{66}$$

Defining the total demand for domestic and foreign non-durables as $C_{X,t}^{TOT}=C_{X,t}+C_{X,t}^B$ with *X= H, F*, it follows from (42), (43), (65) and (66):

$$C_{H,t}^{TOT}=\tau\left(\frac{P_t^H}{P_t^C}\right)^{-\iota_C}C_t^{TOT},\tag{67}$$

$$C_{F,t}^{TOT}=(1-\tau)\left(\frac{P_t^F}{P_t^C}\right)^{-\iota_C}C_t^{TOT},\tag{68}$$

with $C_t^{TOT}=\lambda C_t+(1-\lambda)C_t^B$ being the total amount of non-durable consumption in the domestic country.

*Foreign Country households*

Foreign households face the same utility functions and constraints as domestic households do and they also have the same preference parameters. However, Quint and Rabanal (2014) allow for different values of $\tau$ and $\tau*$.

*7.1.3 Firms*

In both countries, two types of firms exist: final goods producers and intermediate goods producers. Final goods are produced under perfect competition whereas intermediate goods are manufactured under monopolistic competition. The final good uses a continuum of intermediate goods as inputs.

*Final Goods Producers*

Domestic final goods producers use a continuum of domestic intermediate goods indexed $h \in [0, n]$ to produce either non-durable or durable final goods. These goods are homogeneous, and the final goods market is perfectly competitive. Thus, these firms do not make any profits and they regard the price of the final good as well as the prices of the intermediate goods as exogenous. Prices of the final goods are perfectly flexible. Firms producing non-durables can sell their product on the domestic and foreign market whereas durable goods can only be sold on domestic markets.

The production function has the following form:

$$Y_t^k \equiv \left[\left(\frac{1}{n}\right)^{\frac{1}{\sigma_k}} \int_0^n Y_t^k(h)^{\frac{1-\sigma_k}{\sigma_k}}\right]^{\frac{\sigma_k}{1-\sigma_k}}, \text{for } k = \{C, D\}, \tag{69}$$

with $Y_t^k$ denoting the final good, $Y_t^k(h)$ representing domestic intermediate goods and $\sigma_k$ being the price elasticity of intermediate goods. The prices of a domestic non-durable and durable intermediate good are given by $P_t^H(h)$ and $P_t^D(h)$.

Firms that produce non-durables try to maximise their profits given by:

$$\max_{Y_t^C(h)} \pi_t^{C,F} = P_t^H \left[\left(\frac{1}{n}\right)^{\frac{1}{\sigma_C}} \int_0^n Y_t^C(h)^{\frac{\sigma_C-1}{\sigma_C}} dh\right]^{\frac{\sigma_C}{\sigma_C-1}} - \int_0^n Y_t^C(h) P_t^H(h) dh. \tag{70}$$

The first order condition is equal to

$$\frac{\partial \pi_t^{C,F}}{\partial Y_t^C(h)} = \left(\frac{1}{n}\right)^{\frac{1}{\sigma_C}} P_t^H Y_t^C(h)^{\frac{1}{\sigma_C}} \left[\left(\frac{1}{n}\right)^{\frac{1}{\sigma_C}} \int_0^n Y_t^C(h)^{\frac{\sigma_C-1}{\sigma_C}} dh\right]^{\frac{\sigma_C}{\sigma_C-1}} - P_t^H(h) = 0, \tag{71}$$

Inserting (69) into (71) and solving for $Y_t^k(h)$ yields:

$$Y_t^C(h) = \frac{1}{n}\left(\frac{P_t^H(h)}{P_t^H}\right)^{-\sigma_C} Y_t^C. \tag{72}$$

The maximisation problem of the firms producing durables is analogous and leads to the firm's demand function[38]:

$$Y_t^D(h) = \frac{1}{n}\left(\frac{P_t^D(h)}{P_t^D}\right)^{-\sigma_D} Y_t^D. \tag{73}$$

Substituting (72) into (69), it follows

$$Y_t^C = \left[\left(\frac{1}{n}\right)^{\frac{1}{\sigma_C}} \int_0^n \left(\frac{1}{n}\left(\frac{P_t^H(h)}{P_t^H}\right)^{-\sigma_C} Y_t^C\right)^{\frac{1-\sigma_C}{\sigma_C}}\right]^{\frac{\sigma_C}{1-\sigma_C}}, \tag{74}$$

which can be simplified and solved for $P_t^H$ to receive the price level for domestically produced non-durables:

$$P_t^H = \left[\left(\frac{1}{n}\right) \int_0^n (P_t^H(h))^{1-\sigma_C}\right]^{\frac{1}{1-\sigma_C}}. \tag{75}$$

The equivalent procedure yields to the price level for domestically produced durables:

$$P_t^D = \left[\left(\frac{1}{n}\right) \int_0^n (P_t^D(h))^{1-\sigma_D}\right]^{\frac{1}{1-\sigma_D}}. \tag{76}$$

With the given information, the price level for domestic non-durable consumption can be written as

$$P_t^C = [\tau(P_t^H)^{1-\sigma_C} + (1-\tau)(P_t^F)^{1-\sigma_C}]^{\frac{1}{1-\sigma_C}}. \tag{77}$$

*Intermediate Goods Producers*

In the domestic country, *n* intermediate goods producers manufacture one product each. As the intermediate goods are not perfectly substitutable, intermediate goods producers have market

[38] In Quint and Rabanal (2014, p.185), the term $\frac{1}{n}$ is not included in both demand functions. They present the aggregate demand for the intermediate good and not the individual firm's demand. To receive the aggregate demand the single demand has to be multiplied by the population size. Consequently, the term $\frac{1}{n}$ cancels out.

power and the goods are produced under monopolistic competition. The strength of the market power depends on the degree of substitutability, $\sigma_C$ or $\sigma_D$, respectively. Nominal rigidities are introduced following Calvo (1983) as only a fraction $1-\theta_C$ (or $1-\theta_D$) can reoptimise their price in every period. The prices of the other fraction are assumed to be adjusted to the sector's past inflation with parameter $\phi_C$ (or $\phi_D$) measuring the influence (see equation (85)). The intermediate goods are produced with labour being the only production input. The production functions of each sector have the following form:

$$Y_t^C(h) = A_t Z_t^C L_t^C(h), \text{for } h \in [0,n], \tag{78}$$

and $$Y_t^D(h) = A_t Z_t^D L_t^D(h), \text{for } h \in [0,n]. \tag{79}$$

$Z_t^C$ and $Z_t^D$ denote stationary technology shocks which are country and sector-specific. They follow a zero-mean AR(1) process in logs. Moreover, the production function includes a non-stationary technology shock, $A_t$, that hits both countries and both sectors. It follows a unit-root process in logs:

$$log(A_t) = log(A_{t-1}) + \varepsilon_t^A. \tag{80}$$

Due to this shock, the model is non-stationary and the shock "gives a model-consistent way of detrending the data by taking logs and first differences to the real variables that inherit the random walk behaviour" (Quint and Rabanal 2014, p. 186). Furthermore, it can explain co-movement of variables across the two countries which is important from an empirical perspective.

To derive real marginal costs, the following cost minimisation problem is solved for non-durable intermediate goods:

$$\min_{L_t^C(h)} \left(\frac{W_t^C}{P_t^H}\right) L_t^C(h) \ \text{ s.t. } Y_t^C(h) = A_t Z_t^C L_t^C(h), \text{for } h \in [0,n]. \tag{81}$$

The Lagrange function is

$$\min_{L_t^C(h)} \mathcal{L} = \left(\frac{W_t^C}{P_t^H}\right) L_t^C(h) + \lambda_t^{MCC}\Big( Y_t^C(h) - A_t Z_t^C L_t^C(h)\Big), \text{for } h \in [0,n], \tag{82}$$

where $\lambda_t^{MC}$ is equal to the real marginal costs. Deriving the first order-condition and solving for $\lambda_t^{MC}$ yields to:

$$\frac{\partial \mathcal{L}}{\partial L_t^C(h)} = \left(\frac{W_t^C}{P_t^H}\right) - \lambda_t^{MCC} A_t Z_t^C = 0,$$

$$\Leftrightarrow \lambda_t^{MCC} = MC_t^C = \frac{W_t^C / P_t^H}{A_t Z_t^C}. \tag{83}$$

The same procedure leads to the real marginal costs of the durable production:

$$\lambda_t^{MCD} = MC_t^D = \frac{W_t^D / P_t^D}{A_t Z_t^D}. \tag{84}$$

Using (78) and (83) the profit maximisation problem of the non-durable intermediate goods sector (and analogously for the durable sector using (79) and (83)) can be stated as:

$$\max_{P_t^H(h)} \Pi_t^{C,I} = E_t \sum_{k=0}^{\infty} \theta_C^k \Lambda_{t,t+k} \left\{ \left[ \frac{P_t^C(h) \left(\frac{P_{t+k-1}^C}{P_{t-1}^C}\right)^{\phi_C}}{P_{t+k}^C} - MC_{t+k}^C \right] Y_{t+k}^C(h) \right\}$$

$$s.t.\ Y_{t+k}^C(h) = \left[\frac{P_t^H(h)}{P_{t+k}^H} \left(\frac{P_{t+k-1}^H}{P_{t-1}^H}\right)\right]^{-\sigma_C} Y_{t+k}^C, \tag{72}$$

where $Y_{t+k}^C(h)$ represents future demand of the final goods sector for intermediate good $h$. The stochastic discount factor is $\Lambda_{t,t+k} = \beta^k \frac{\delta_{t+k}}{\delta_t}$, with $\delta_t$ representing the marginal utility of labour. Each intermediate goods producing firm tries to maximise sum of future revenues minus marginal costs.

Inserting the constraint into the profit function, taking the first derivate and solving for $\frac{\hat{P}_t^H(h)}{P_t^H}$ leads to

$$\frac{\hat{P}_t^H(h)}{P_t^H} = \frac{\sigma_C}{\sigma_C - 1} E_t \left[ \frac{\sum_{k=0}^{\infty} \beta^k \theta_C^k \delta_{t+k} \left( \prod_{s=1}^{k} \frac{\left(P_{t+s-1}^C / P_{t+s-2}^C\right)^{\sigma_C}}{P_{t+s}^C / P_{t+s-1}^C} \right)^{-\sigma_C} MC_{t+k}^C Y_{t+k}^C}{\sum_{k=0}^{\infty} \beta^k \theta_C^k \delta_{t+k} \left( \prod_{s=1}^{k} \frac{\left(P_{t+s-1}^C / P_{t+s-2}^C\right)^{\sigma_C}}{P_{t+s}^C / P_{t+s-1}^C} \right)^{1-\sigma_C} Y_{t+k}^C} \right], \tag{86}$$

with $\hat{P}_t^H(h)$ representing the optimal price that can be set in period *t*. One can drop the h as the maximisation problem is the same among all firms that can optimise their price. Thus, $\hat{P}_t^H(h)$ is equal to $\hat{P}_t^H$. The result for the durable sector is the same as (86) (simply change *H* and *C* to *D*). Using the information above, the price indices for domestically produced non-durables (75) and durables (76) can be simplified. The price level of the firms that cannot adjust in period t is equal to the last period's price level adjusted by $\left(\frac{P_{t-2}^H}{P_{t-1}^H}\right)^{-\sigma_C}$:

$$P_t^C = \left[\frac{1}{n}\left(n(1-\theta_C)\left(\hat{P}_t^H\right)^{1-\sigma_C} + n\theta_C\left[P_{t-1}^H\left(\frac{P_{t-2}^H}{P_{t-1}^H}\right)^{-\sigma_C}\right]^{1-\sigma_C}\right)\right]^{\frac{1}{1-\sigma_C}},$$

$$\Leftrightarrow P_t^C = \left[\left((1-\theta_C)\left(\hat{P}_t^H\right)^{1-\sigma_C} + \theta_C\left[P_{t-1}^H\left(\frac{P_{t-1}^H}{P_{t-2}^H}\right)^{\sigma_C}\right]^{1-\sigma_C}\right)\right]^{\frac{1}{1-\sigma_C}} \tag{86}$$

and $$P_t^D = \left[\left((1-\theta_D)\left(\hat{P}_t^D\right)^{1-\sigma_D} + \theta_D\left[P_{t-1}^D\left(\frac{P_{t-1}^D}{P_{t-2}^D}\right)^{\sigma_D}\right]^{1-\sigma_D}\right)\right]^{\frac{1}{1-\sigma_D}}. \tag{87}$$

*Foreign goods producers*
In the foreign country, final and intermediate goods producers face equivalent maximisation problems.

Combining the results from the domestic and the foreign country, one can derive the union-wide CPI index which is defined as the geometric average of the two countries' CPI indices. The union-wide real GDP is defined in the same way:

$$P_t^{EMU} = (P_t^C)^n (P_t^{C*})^{1-n}, \tag{89}$$

and $$Y_t^{EMU} = (Y_t)^n (Y_t^*)^{1-n}, \tag{90}$$

where the * denotes foreign variables and the national GDPs are denoted in price units of non-durables:

$$Y_t = Y_t^C + Y_t^D \frac{P_t^D}{P_t^C}, \tag{91}$$

$$Y_t^* = Y_t^{C*} + Y_t^{D*} \frac{P_t^{D*}}{P_t^{C*}}. \tag{92}$$

*7.1.4 Closing the Model*

To close the model derivation, the market clearing conditions are stated. These conditions are presented in aggregate quantities and thus, each per-capita quantity is multiplied by the relevant country's population size.

Supply of intermediate goods equals their demand by final goods producers. That is

$$Y_t^C(h) = \left(\frac{P_t^H(h)}{P_t^H}\right)^{-\sigma_C} Y_t^C, \tag{93}$$

and $$Y_t^D(h) = \frac{1}{n}\left(\frac{P_t^D(h)}{P_t^H}\right)^{-\sigma_D} Y_t^D. \tag{94}$$

The aggregate production of non-durable final goods is equal to the total demand which consists of the demand by domestic savers $C_{H,t}$ and borrowers $C_{H,t}^B$ as well as the demand of foreign savers $C_{H,t}^*$ and borrowers $C_{H,t}^{B*}$. The foreign demand represents the amount of domestic exports.

$$nY_t^C = n\left[\lambda C_{H,t} + (1-\lambda)C_{H,t}^B\right] + (1-n)\left[\lambda C_{H,t}^* + (1-\lambda)C_{H,t}^{B*}\right]. \tag{95}$$

As durable goods cannot be exported, the production of durables equals the residential investment by domestic savers and borrowers:

$$nY_t^D = n[\lambda I_t + (1-\lambda)I_t^B]. \tag{96}$$

Moreover, the aggregate labour supply for both sectors by savers and borrowers has to be equal to the total hours worked in both sectors:

$$\int_0^n L_t^k(h)dh = \lambda \int_0^n L_t^{k,j}dj + (1-\lambda)\int_0^n L_t^{k,B,j}dj, \text{ for } k=C, D \tag{97}$$

Regarding the credit market, the sum of the domestic savers' deposits and foreign bonds, combined with the restriction implemented by the macroprudential policy tool $\eta_t$, must equal the sum of credit granted to borrowers:

$$\frac{n\lambda(S_t - B_t)}{\eta_t} = n(1-\lambda)S_t^B. \tag{98}$$

Besides, the sum of domestic and foreign bonds must equal zero:

$$n\lambda B_t + (1-n)\lambda B_t^* = 0. \tag{99}$$

*Monetary Policy*

The interest rate or risk-free rate is set by the central bank as mentioned earlier. As both countries are in a currency union, the central bank's interest rate setting directly affects the home country, as the domestic deposit rate equals the risk-free rate. It influences the foreign country through the mechanism described in equation (19).

The interest rate reacts to deviations of the union-wide inflation rate from its steady-state value and to the union-wide real output growth. Moreover, it depends on the last-period's interest rate. This can be interpreted as a preference for interest-rate-smoothing over time. The mechanism is also hit by an exogenous monetary policy shock.

The mentioned causal relationship between the interest rate and the different variables can be stated formally according to this adapted Taylor-Rule:

$$R_t = \left[\bar{R}\left(\frac{P_t^{EMU}/P_{t-1}^{EMU}}{\bar{P}_G^{EMU}}\right)^{\gamma_\pi}\left(\frac{Y_t^{EM}}{Y_{t-1}^{EMU}}\right)^{\gamma_y}\left(\frac{S_t^{B,EMU}}{S_{t-1}^{B,EMU}}\right)^{\gamma_{cg}}\right]^{1-\gamma_R} R_{t-1}{}^{\gamma_R}\, exp(\varepsilon_t^m), \tag{100}$$

with $\bar{R}$ being the interest rate's steady-state value and $P_t^{EMU}$ the value of the union-wide Consumer Price Index. The steady-state value of the price index growth rate is denoted by $\bar{P}_G^{EMU}$. The parameters $\gamma_\pi, \gamma_y$ and $\gamma_R$ represent the weights that the central bank applies to inflation, output-growth and interest-rate-smoothing. When discussing the 'leaning against the wind' approach, this simple Taylor rule is appended with the EMU-wide nominal credit growth $S_t^{B,EMU}/S_{t-1}^{B,EMU}$. The parameter $\gamma_{cg}$ is measuring the strength of the interest rate's reaction to nominal credit growth. For $\gamma_{cg} = 0$ credit growth cancels out and a non-altered Taylor Rule setting can be studied.

I model the union-wide amount of credit as the geometric average of the two countries' credit amounts weighted by the country size[39]:

$$S_t^{B,EMU} = (S_t^B)^n (S_t^{B*})^{1-n}. \tag{101}$$

[39] Unfortunately, Quint and Rabanal (2014a) do not present an analytical expression of union-wide credit growth. Although using the geometric average is in line with the other given union-wide definitions, it is a possible deviation from the original framework.

*Macroprudential Policy*

In equation (6) is shown that macroprudential policy influences the credit market through the balance sheets of financial intermediaries. The case in which $\eta_t$ equals one represents the scenario without macroprudential policy. The exact value of $\eta_t$ is set using the following equation:

$$\eta_t = (\Upsilon_t)^{\gamma_\eta}. \tag{102}$$

In this formula, $\Upsilon_t$ denotes an indicator that represents deviations from steady-state values of a certain model variable. The macroprudential policy tool reacts to $\Upsilon_t$ whereby the strength of this reaction depends on $\gamma_\eta$ which is set in order to minimise a loss or to maximise a welfare function. For the foreign country, $\eta_t^*$ is designed analogously. Quint and Rabanal (2014a) force $\gamma_\eta$ and $\gamma_\Upsilon^*$ to be equal to model macroprudential policy conducted by union-wide institution such as the central bank. I assume that using (89) with an indicator variable that represents a union-wide variable can also model this situation. In this case, $\eta_t$ is applied in both countries. However, this paper adopts the first approach. Generally, the current discussion offers several indicators that can be used instead.

## 7.2 Computational Remarks

The whole computational work was done using Dynare and Matlab. The Dynare mod-file was taken from the Macroeconomic Model Data Base (2021) and then adapted to fit my own needs.

*Optimisation Routine*

I searched numerically for the optimal coefficients that minimise central bank losses. The parameters are not derived analytically. Instead, the model is simulated for every possible tuple of the parameters, $\gamma_\pi$, $\gamma_y$, $\gamma_{cg}$, $\gamma_R$ and $\gamma_\eta$, and the tuple that lead to the minimum loss is chosen to be the optimal one[40]. The parameters are optimised to the second decimal place. The best procedure would be to construct a loop with an incremental parameter increase of 0.01. Due to limitations in the computing power, this method had to be adapted[41]. Instead, an approach was implemented that contains three steps: first, the loop was executed with incremental parameter increases of 0.2 and the initial parameter bounds specified in section 3.2. A resulting

[40] Conditional on the policy mix that is optimised, not every of the five parameters is included in the procedure.

[41] Extrapolated run-times of other simulations, the initial procedure would have taken more than one week in the case of an altered Taylor Rule with macroprudential policy.

representative optimal parameter from the first stage was denoted $P_1^*$. Second, the loop was carried out again but this time with incremental increases of 0.1. The parameters' lower (upper) bounds were defined as $P_1^* - 0.2$ ($P_1^* + 0.2$). Again, a resulting optimal parameter was denoted $P_2^*$. Third, the parameter bounds were defined as $P_2^* - 0.1$ and $P_2^* + 0.1$, respectively, and the loop was executed with incremental increases of 0.01. Finally, the optimal parameters were extracted from the third looping process.

However, this method can deliver false results in the presence of multiple local minima. To avoid such problems the optimisation process should be started from different initial values which was done in this analysis. Nevertheless, the welfare gains of stage two and three of the optimisation cycle were often very limited as shown in table 6. Thus, the weaknesses of the approach are limited to a certain extend.

**Table 6:** Stage Results of Grid Search

| | Incremental Increase | $\gamma_\pi$ | $\gamma_y$ | $\gamma_R$ | $\gamma_{cg}$ | $\gamma_\eta$ | Welfare Gain |
|---|---|---|---|---|---|---|---|
| | | | All Shocks | | | | |
| Estimated Taylor Rule | | 1.5579 | 0.2023 | 0.8016 | - | - | Baseline |
| Simple Taylor Rule | 0.2 | 5.00 | 3.80 | 0.20 | - | - | 6.024 % |
| | 0.1 | 5.00 | 3.70 | 0.20 | - | - | 6.027 % |
| | 0.01 | 5.00 | 3.71 | 0.23 | - | - | 6.027 % |
| Altered Taylor Rule | 0.2 | 5.00 | 3.80 | 0.00 | 1.20 | - | 7.948 % |
| | 0.1 | 5.00 | 3.70 | 0.00 | 1.20 | - | 7.955 % |
| | 0.01 | 5.00 | 3.70 | 0.00 | 1.23 | - | 7.955 % |
| Simple Taylor Rule and Macroprudential Policy | 0.2 | 5.00 | 3.80 | 0.20 | - | 0.00 | 6.024 % |
| | 0.1 | 5.00 | 3.70 | 0.20 | - | 0.10 | 6.085 % |
| | 0.01 | 5.00 | 3.72 | 0.22 | - | 0.09 | 6.088 % |
| Altered Taylor Rule and Macroprudential Policy | 0.2 | 5.00 | 3.80 | 0.00 | 1.20 | 0.00 | 7.948 % |
| | 0.1 | 5.00 | 3.70 | 0.00 | 1.20 | 0.00 | 7.955 % |
| | 0.01 | 5.00 | 3.67 | 0.00 | 1.24 | 0.03 | 7.961 % |
| | | | Preference shocks | | | | |
| Estimated Taylor Rule | | 1.5579 | 0.2023 | 0.8016 | - | - | Baseline |
| Simple Taylor Rule | 0.2 | 2.20 | 5.00 | 0.20 | - | - | 35.352 % |
| | 0.1 | 2.30 | 5.00 | 0.30 | - | - | 35.380 % |
| | 0.01 | 2.27 | 5.00 | 0.28 | - | - | 35.386 % |
| Altered Taylor Rule | 0.2 | 2.00 | 5.00 | 0.00 | 2.20 | - | 40.067 % |
| | 0.1 | 2.10 | 5.00 | 0.00 | 2.30 | - | 40.072 % |
| | 0.01 | 2.09 | 5.00 | 0.00 | 2.27 | - | 40.072 % |
| Simple Taylor Rule and Macroprudential Policy | 0.2 | 2.20 | 5.00 | 0.40 | - | 0.20 | 35.460 % |
| | 0.1 | 2.30 | 5.00 | 0.30 | - | 0.20 | 35.494 % |
| | 0.01 | 2.26 | 5.00 | 0.30 | - | 0.18 | 35.499 % |
| Altered Taylor Rule and Macroprudential Policy | 0.2 | 2.00 | 5.00 | 0.00 | 2.20 | 0.00 | 40.067 % |
| | 0.1 | 2.00 | 5.00 | 0.00 | 2.30 | 0.00 | 40.072 % |
| | 0.01 | 2.09 | 5.00 | 0.00 | 2.27 | 0.00 | 40.072 % |

**Table 6:** Continued

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| Financial Shocks | | | | | | | |
| Estimated Taylor Rule | | 1.5579 | 0.2023 | 0.8016 | - | - | Baseline |
| Simple Taylor Rule | 0.2 | 5.00 | 5.00 | 0.60 | - | - | 79.130 % |
| | 0.1 | 5.00 | 5.00 | 0.60 | - | - | 79.130 % |
| | 0.01 | 5.00 | 5.00 | 0.61 | - | - | 79.137 % |
| Altered Taylor Rule | 0.2 | 5.00 | 5.00 | 0.60 | 0.20 | - | 81.398 % |
| | 0.1 | 5.00 | 5.00 | 0.70 | 0.30 | - | 81.842 % |
| | 0.01 | 5.00 | 5.00 | 0.68 | 0.32 | - | 81.890 % |
| Simple Taylor Rule and Macroprudential Policy | 0.2 | 5.00 | 5.00 | 0.60 | - | 0.20 | 79.305 % |
| | 0.1 | 5.00 | 5.00 | 0.60 | - | 0.10 | 80.474 % |
| | 0.01 | 5.00 | 5.00 | 0.58 | - | 0.09 | 80.492 % |
| Altered Taylor Rule and Macroprudential Policy | 0.2 | 5.00 | 5.00 | 0.60 | 0.20 | 0.00 | 81.398 % |
| | 0.1 | 5.00 | 5.00 | 0.60 | 0.30 | 0.10 | 82.431 % |
| | 0.01 | 5.00 | 5.00 | 0.65 | 0.29 | 0.07 | 82.629 % |
| Technology Shocks | | | | | | | |
| Estimated Taylor Rule | | 1.5579 | 0.2023 | 0.8016 | - | - | Baseline |
| Simple Taylor Rule | 0.2 | 1.20 | 0.00 | 0.80 | - | - | 9.382 % |
| | 0.1 | 1.10 | 0.00 | 0.80 | - | - | 10.967 % |
| | 0.01 | 1.01 | 0.00 | 0.79 | - | - | 12.338 % |
| Altered Taylor Rule | 0.2 | 1.20 | 0.00 | 0.80 | 0.00 | - | 9.382 % |
| | 0.1 | 1.10 | 0.00 | 0.80 | 0.00 | - | 10.967 % |
| | 0.01 | 1.01 | 0.00 | 0.79 | 0.00 | - | 12.338 % |
| Simple Taylor Rule and Macroprudential Policy | 0.2 | 1.20 | 0.00 | 0.80 | - | 0.00 | 9.382 % |
| | 0.1 | 1.10 | 0.00 | 0.80 | - | 0.00 | 10.967 % |
| | 0.01 | 1.01 | 0.00 | 0.79 | - | 0.00 | 12.338 % |
| Altered Taylor Rule and Macroprudential Policy | 0.2 | 1.20 | 0.00 | 0.80 | 0.00 | 0.00 | 9.382 % |
| | 0.1 | 1.10 | 0.00 | 0.80 | 0.00 | 0.00 | 10.967 % |
| | 0.01 | 1.01 | 0.00 | 0.79 | 0.00 | 0.00 | 12.338 % |